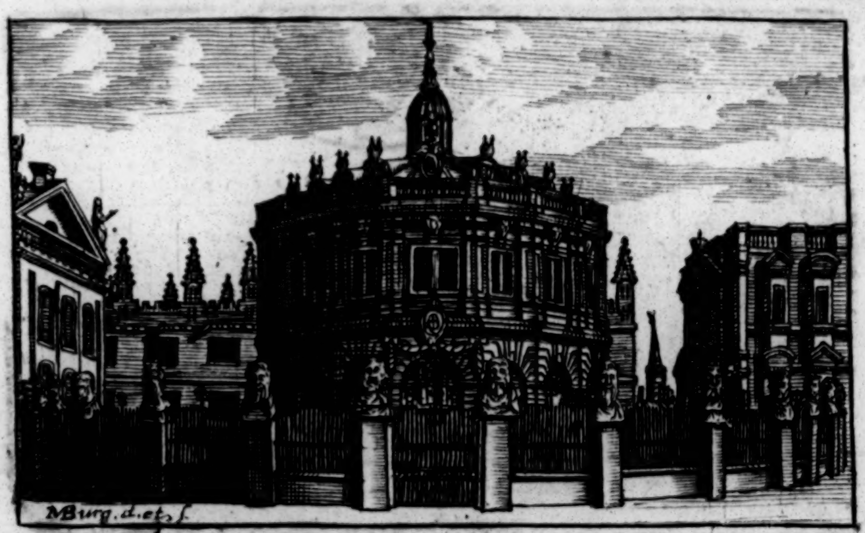
SECTIONUM CONICARUM *ELEMENTA* NOVA METHODO DEMONSTRATA.

Authore JACOBO MILNES, A. M. Rectore de INGESTRE in Agro STAFFORDIENSI.

EDITIO TERTIA
Novâ & SEXTA *Parte auctior.*



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STUDIOSÆ JUVENTUTIS

OPUSCULUM HOC

Humillime D.

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PRÆFATIO.

EN tibi, Lector candide, Tertiam horum Elementorum Editionem; in quâ licèt (ſicut in ſecundâ) præſcriptos mihi primitùs (quod ad operis molem attinet) limites tranſcenderim, à principali tamèn, quem mihi ab initio propoſui, ſcopo libens non aberravi, Ut (*nempè*) præcipuas Sectionum Conicarum proprietates quam poſſem breviter & dilucide demonſtratas darem.

Hoc propoſitum an aliquâ omnino, aut quâ ex parte, quo etiam modo & ratione, conſecutus fuerim, cum tu melius ex ipſa lectione, quam noſtro quovis præloquio, poſſis diſcere, vix alia cauſa fuit cur te hic in limine detinerem, quàm quod nefas ducerem hæc in publicum emittere, niſi ſimul per quos in iiſdem concinnandis profecerim ingenuè profiterer, gratuſque agnoſcerem.

Scias itaque hujus opuſculi materiem ex Cl.V. Ph. de la Hire *opere maximo magna ex parte deſumptam eſſe; & licet quoad operis compagem, præſertim in primis Sectionum affectionibus eliciendis, pro diverſi inſtituti ratione, diverſæ longè methodo inſtiterim, in cæteris tamen me (quantum licuit) veſtigia ejus libenter preſſiſſe, & demonſtrationes plerumque integras adhibuiſſe.*

Quæ continentur in propp. 35. & 36. p. 1. & earum corollariis, & prop 4. p. 5. mihi ſuppeditavit Incomparabilis viri D. Iſ. Newton Principiorum Liber 1. *immutatâ paululùm tam enuntiationis quam demonſtrationis formâ, & propoſito noſtro accommodatâ.*

Sectionis Conicæ proprietatem illam quam in prop. 6. p. 4. reperies, abſque demonſtratione mihi impertiit V.

Cl. Dav. Gregorius *M. D. & in Cathedra* Saviliana *Aſtronomiæ profeſſor, cum primam hujuſce tractatus editionem molirer ei inſerendam.*

Propoſitionis 11. *p.* 2. *demonſtrationem, elegantiorem multò quàm alibi mihi reperire contigit, mecum olim communicavit V. Cl.* Jo. Keill *poſtea M. D. &* Gregorii *in eadem cathedra tandem ſucceſſor. Qui etiam communicavit ea quæ in corollario* 2. *prop.* 4. *& prop.* 5. *p.* 6. *ſed noſtro modo demonſtrata, exhibentur, quæque V. Cl. Tranſactionibus Philoſophicis poſtea inſeruit.*

Præter hæc jam particulatim enumerata, cætera omnia (ſive ipſæ ſectionum proprietates, ſive demonſtrationes ſpeciales) quæ apud Cl. de la Hire *non extant, ex proprio penu ſunt, quæque mihi argumentum hoc preſſiùs tractanti ſeſe obvia tulere; Ea autem, vel ob nativam dignitatem, vel uſum quem præ ſe ferre viſa ſunt, non te celanda exiſtimavi, an alibi extarent prorſus ignarus.*

Multa autem paſſim in hoc opere me inter corollaria conjeciſſe invenies, quæ ipſis theorematibus nec uſu, nec dignitate cedunt; quod ut mihi compendia ſectanti quoties (ob conſequentiæ facilitatem) cum fructu fieri id poſſet, licuiſſe ratus ſim, ita, ne propterea de eorum aliquibus tibi juſto minor ſederet opinio, te ejus hîc monendum eſſe oportuit.

Quæ dixi hactenus totum opus in genere reſpiciunt. Quod vero ad Sextam partem, quam hæc Editio primùm exhibet, ſpeciatim attinet, conſtat ea ex proprietatibus (quantum ego equidem ſcio) omnino novis, paucis admodum exceptis, quas prius notas, ſuccinctiore quam hactenus modo demonſtratas dedi. His, cùm uſum non contemnendum præſtare poſſe viderentur, nolui lector ut careres; & licet varii generis eſſent, malui ut quam hîc ſortitæ ſunt partem ſeorſim conficerent, quàm ſingulas quaſque locis propriis inſerendo receptum propoſitionum & figurarum ordinem turbare.

Atque hîc, lector, te vix ſalutatum valere jubeo, hoſque noſtros conatus candori tuo commendo. Si in ipſam *veritatem nuſquam peccaverim, levioribus admiſſis (ſpero) facilè ignoſces. Quod Elementa hæc manca & imperfecta ſint, ſi vitio vertas, id cum aliis omnibus compendiosè ſcriptis commune habent. Quos denique labores (tui gratiâ) in iis elucubrandis devorare coactus ſim, ut ipſe inexpertus non facile intelliges, ita nolim ex parva libelli mole dijudices; horum certè (quot quantive fuerint) dum tibi proſim, me nunquam pigebit; imo ſemper juvabit meminiſſe.*

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Quæ addenda & corrigenda ſunt inveniet lector poſt finem libri.

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PARS

PARS I.

SECTIO I.

Definitiones.

I. SI recta linea A E per punctum quodvis A extra circuli B D E C planum poſitum utrinque infinite extenſa, manente puncto A, per totam circuli peripheriam circumagatur; Binæ hujuſmodi motu genitæ ſuperficies ſigillatim *Conicæ ſuperficies* appellantur. *Fig.* 1, 2.

II. Conjunctim vero *Superficies ad verticem oppoſitæ*; vel ſimpliciter *Superficies oppoſitæ.*

III. Punctum A *Vertex* dicitur.

IV. Circulus B D E C *Baſis.*

V. Recta A C per verticem A & baſis centrum C utrinque infinite producta *Axis.*

VI. Solidum ſuperficie conicâ & baſi contentum *Conus* dicitur.

VII. Et quidem, ſi axis ad baſim rectus fuerit, *Conus Rectus.* 1.

VIII. Si inclinatus, *Conus Scalenus.* 2.

IX. Communis plani alicujus cum ſuperficie conicâ interſectio, *Sectio Conica* dicitur.

Corollarium def. 1. Recta per punctum quodvis in utrâvis ſuperficie & verticem ducta, tota eſt in eâdem ſuperficie; Productaque ultra verticem eſt in ſuperficie oppoſita.

Propoſitio I. Theorema I.

Recta linea A E per verticem A ducta ad quodvis punctum E intra utramvis ſuperficierum oppoſitarum, 3.

quantumvis utrinque producta, intra ſuperficies conicas continetur. Rectaque per verticem A ducta ad quodlibet punctum G extra utramque ſuperficiem, quantumvis producta, extra utramque ſuperficiem manet.

Hæc propoſitio clarior eſt quam ut demonſtratione egeat.

Prop. II. Theor. II.

4. *Si per Coni verticem A tranſeat planum A B C, ſecans utcunque Conicas ſuperficies; Eaſdem in duabus rectis A B, A C ſecabit.*

Sit interſectio plani ſecantis & plani baſis recta B C, & à punctis B, C, in quibus hæc occurrit baſis peripheriæ, ad verticem A ductæ intelligantur rectæ B A, C A; Erunt hæ tum in plano ſecante, tum (per *Coroll.* præced.) in ſuperficiebus Conicis, hoc eſt, ſunt plani & ſuperficierum interſectiones. Porro ab A ad punctum quodvis F in baſis peripheria, à B & C diverſo, ducatur recta A F; Erit hæc ubique extra planum A B C; hoc eſt, planum A B C ſuperficiebus Conicis ſolummodo in rectis A B, B C occurrit.

Prop. III. Theor. III.

5. *Recta linea E D conjungens bina quæcunque puncta E, D, in eadem conica ſuperficie ſumpta, modò non ſint in eadem recta per verticem, tota cadit intra ſuperficiem conicam, producta vero utrinque extra eandem; & neutri ſuperficierum oppoſitarum amplius*
6. *occurrit. Contra, Recta E D conjungens bina quæcunque puncta E, D, in oppoſitis ſuperficiebus, modò non in eadem recta per verticem, cadit extra utramque ſuperficiem, productaque utrinque intra utramque tranſit, neutrique earum amplius occurrit.*

Nam ductis rectis A E, A D, & productis (ſi opus) in C, B, Planum per A E C, A D B, in quo D E ſita eſt;

eſt, non occurrit ſuperficiei vel ſuperficiebus oppoſitis niſi in rectis AEC, ADB; unde recta DE non
occurrit ipſis niſi in punctis D, E. In priore vero caſu 5
planum anguli DAE, proindeque recta DE, eſt intra
ſuperficiem conicam; productaque utrinque extra eandem cadet. In poſteriore planum anguli DAE eſt ex- 6
tra utramque ſuperficierum oppoſitarum, proindeque & recta DE; producta vero utrinque intra utramque tranſibit.

Si puncta D, E ſint in eadem recta per verticem, erit recta ipſas conjungens (per *Coroll.* præced.) in ipſis ſuperficiebus conicis.

Prop. IV. Theor. IV.

Si circulum qui baſis eſt Coni contingat recta D E in D,
à Coni autem vertice A ad contactum D agatur recta 7.
AD; Planum ADE per utramque rectam productum ſuperficiei conicæ ſolummodo in recta AD occurrit. Hujuſmodi vero occurſus Contactus *dicitur.*

Recta AD tum ſuperficiei conicæ, tum plano ADE communis eſt; Cùm vero, præter D, quodvis punctum F in baſeos peripheria ſit extra rectam DE, Erit etiam, præter DA, quævis alia recta FA in ſuperficie Conica extra planum ADE; unde liquet propoſitum.

Coroll. 1. Hinc & ex def. 1. liquet planum ADE productum ſuperficiem oppoſitam in DA producta contingere.

Coroll. 2. Hinc etiam patet methodus Ducendi planum quod ſuperficiem Conicam in data recta AD contingat. Ductâ nempe in plano baſeos rectâ DE peripheriam ejus in D contingente, Actóque per AD, DE plano ADE, propoſito ſatisfiet.

Coroll. 3. Præter ADE, aliud planum ſecundum AD ſuperficiem conicam non contingit. Nam interſectio ejus cum baſi hujus peripheriam ſecabit; unde & planum ipſum ſuperficiem conicam ſecabit, hoc eſt, non continget.

Prop. V. Problema I.

8. *Per rectam AC, per coni verticem A extra ſuperficiem ejus utcunque ductam, Planum ducere quod ſuperficiem conicam contingat.*

Per AC agatur utcunque planum, ſecans baſim in GH, Cui recta AC (cum in eodem ſit plano) occurret, vel erit parallela.

1°. Occurrat in E; A puncto E baſeos peripheriam contingat ex utravis parte recta ED; Connexâ AD, Erit ADE planum quæſitum.

Tranſit enim per AC, & (per præced.) ſuperficiem conicam contingit.

9. 2°. Sit AC parallela GH; Biſectâ GH in M, erectâque, ex utravis parte, ad GH perpendiculari MD, factâque DE parallelâ GH, & connexâ AD; Erit iterum ADE planum quæſitum.

Nam (ob AC || GH || DE) erit AC in plano ADE; Continget vero DE baſeos peripheriam in D; unde (per præced.) planum ADE Conicam ſuperficiem continget.

Coroll. Liquet bina tantum plana per AC ſuperficiem conicam contingere; Ex utraque ſcilicet parte plani AGH unum.

Prop. VI. Theor. V.

10. *Recta quævis ED, rectæ cuivis AB per verticem A in ipſa ſuperficie conica ductæ, parallela, (modò non ſit in plano ſuperficies in AB contingente) Uni tantùm ſuperficierum oppoſitarum, idque in unico puncto E, occurrit; Hoc eſt, ex una parte, tota eſt extra utramque ſuperficiem; Ex altera, tota intra illam continetur in qua punctum E ſitum eſt.*

Planum per parallelas AB, ED, ſecet ſuperficies in cAC; Hoc planum BAC, in quo recta ED ſita eſt, non occurrit ſuperficiebus, niſi in AB, cAC; Rectaque E D (ob paralleliſmum) non occurrit rectæ A B; Occurrit tamen neceſſariò rectæ c A C alicubi in E, manetque ex una parte puncti E intra angulum B A C, ex altera intra angulum qui eſt huic deinceps; Unde &c.

Coroll. Liquet planum per D E, plano ſecundum A B ſuperficiem conicam tangenti, parallelum, ſuperficiei oppoſitæ non occurrere.

Prop. VII. Theor. VI.

Omnis recta E F, rectæ cuivis A D per verticem A
ductæ, atque intra ſuperficies conicas cadenti, paral- 11.
lela, utrique ſuperficiei oppoſitæ occurrit alicubi in
E, F. Omniſque recta E F, per punctum quodvis 12.
E in utravis ſuperficie ducta, rectæ cuivis per verti-
cem A D extra ſuperficies cadenti, parallela, Eidem
ſuperficiei occurrit denuò in F; Excepto duntaxat
caſu ubi punctum E fuerit in una rectarum ſecun-
dum quas plana per A D ſuperficies contingunt.

Per A D & rectam E F tranſeat planum, ſecans conicas ſuperficies in B A b, c A C, quod ſemper fieri poſſe, niſi in caſu memorato, plus ſatis manifeſtum eſt; Cum A D cadat intra angulum B A C, vel B A c, Recta E F quæ eſt ipſi A D parallela, & in eodem cum ipſa plano, occurret neceſſariò utrique A B, A C alicubi in E, F; Eritque in priori quidem caſu occurſuum alter F in A B producta, hoc eſt, erunt E, F, in ſuperficiebus oppoſitis.

Sin punctum E, in poſteriori caſu, ſit in recta ſe-
cundum quam planum per A D ſuperficies conicas con- 13.
tingit, manifeſtum eſt planum per A D & punctum E
conicas ſuperficies non ſecare, & rectam E F || A D
eſſe in plano ſuperficies conicas ſecundum rectam
A E C contingente.

Coroll. 1. Hinc & per prop. 3. ulterius liquet Rectam 11.
E F, niſi in punctis E, F, neutri ſuperficierum oppoſi- 12.
tarum amplius occurrere.

Coroll. 2. Omne planum per hujuſmodi rectam E F, 11.
in

12. in priori caſu, ſecabit utramque ſuperficiem; Et in poſteriori, ſi per A D tranſeat planum quodvis, extra utramque ſuperficiem cadens, planum per E F huic parallelum, ſuperficiem conicam, in qua punctum E ſumptum eſt, ſecabit ex omni parte; ſuperficiei tamen oppoſitæ non occurret.

Prop. VIII. Theor. VII.

14. *Si alterutra ſuperficierum oppoſitarum plano ſecetur plano baſeos parallelo, Erit facta ſectio E G F L Circuli circumferentia.*

Secetur Conus utcunque per axem duobus planis, facientibus triangula A B D, A I K, quæ (ſi opus producta) occurrant plano Sectionis in G H L, E H F; Ob plana parallela, erunt triangula A H F, A C K, uti etiam A H L, A C D ſimilia: Quare
AH : AC : : HL : CD
& AH : AC : : HF : CK; unde
HL : HF : : CD : CK. Eſt vero C baſeos centrum, Unde C D = C K, ergo & H L = H F. Pari modo oſtendetur G H = H F; & ſimiliter in quavis alia interſectione plani E G F L cum plano per axem. Eſt ergo ſectio E G F L circuli circumferentia, cujus centrum H.

Prop. IX. Theor. VIII.

15. *Si Conus ſcalenus A B C L ſecetur plano per axem, & ad baſim recto, faciente triangulum A B L; Reſectoque utcunque ab angulo B A L triangulo A D I, triangulo A L B ſimili, ſed ſubcontrarie poſito, (i. e. ut ſit Ang. A L B = A D I) ſecetur iterum Conus plano per D I, ad trianguli A B L planum recto, Erit facta in ſuperficie conica ſectio D K I F circuli circumferentia. Dicitur autem D K I F* Sectio Subcontraria.

Per ipſius D I punctum quodvis G agatur recta E G H, ipſi B L parallela, Per quam tranſeat planum ad planum trianguli rectum, proindeque baſi parallelum, faciens (per præced.) in ſuperficie conica circuli circumferentiam FEKH, planumque per DI ſecans in recta FGK; Erit (propter planum per axem) recta EH circuli FEKH diameter; Et (propter utriuſque ſectionis planum ad planum trianguli rectum) erit eorum interſectio KGF ad utramque DG, EG perpendicularis; Eſt itaque (propter circulum) $EG \times GH = GFq = GKq$; Sunt vero (propter Ang. $AID = AEH$, & $DGE = HGI$) triangula DGE, HGI ſimilia; Unde $DG : GH :: EG : GI$; Ergo $DG \times GI = EG \times GH =$ (prius) $GFq = GKq$; Sunt ergo puncta F, K ad circuli peripheriam, cujus diameter DI. Idemque erit de quovis alio puncto in recta DI. Eſt igitur DKIF Circuli circumferentia.

Prop. X. Theor. IX.

Si in ſuperficie conicâ, a plano quovis baſi non parallelo, fiat ſectio DFIK, quæ ſit circuli circumferentia, Erit eadem ſectio ſubcontraria. 15.

Planum baſi parallelum faciat circulum EKHF, ſecans planum circuli DFIK in KGF; ſitque circuli EKHF diameter EGH ad KGF perpendicularis; Tranſibit hæc per axem, eritque $KG = GF$. Per rectam EGH, & coni axem tranſeat planum, ſecans planum circuli DFIK, in DGI, & baſim in $BL \parallel EH$; Erit (ob circulos) $EG \times GH = GKq = GFq = DG \times GI$; Ergo $EG : DG :: GI : GH$; Unde Triangula DGE, HGI ſimilia. & ang. $DEG = HIG$. Si jam manente plano per axem ABL prius ducto, intelligantur quotcunque circuli EKHF à planis baſi parallelis facti; Erunt horum planorum cum plano per axem interſectiones EGH, circulorum EKHF diametri; & (ob planorum paralleliſmum, & rectam EGH, ad FGK prius perpendicularem) ad reſpectivas eorum interſectiones FGK cum plano circuli DFIK perpendiculares, eaſque ideo biſecabunt; Hoc eſt, recta

DI (quæ eſt in eodem plano per axem) eas omnes biſecabit, & propterea eſt diameter, & ad eaſdem perpendicularis. Recta itaque FGK ad utramque DI, HE perpendicularis eſt; proindeque planum per axem tam ad planum circuli EFHK, (hoc eſt ad baſim,) quam ad planum circuli DFIK rectum eſt. Unde liquet propoſitum.

Coroll. Hinc niſi Sectio conica vel ſit ſectio ſubcontraria, vel fiat à plano baſi parallelo, non erit circuli circumferentia.

Lemma 1.

16. *Si duorum planorum ABE, BCF, cum plano aliquo tertio, interſectiones AD, CF ſint parallelæ; Erit etiam mutua ipſorum interſectio BE ipſis AD, CF parallela.*

Secentur utcunque hæc tria plana à duobus planis inter ſe parallelis, quorum interſectiones conſtituant triangula ACB, DEF; Erunt (ob plana parallela) trianguli ABC latera AC, AB, BC, trianguli DEF lateribus DF, DE, EF, reſpective parallela; Unde anguli parallelis lateribus contenti erunt æquales: Sed (ob parallelas rectas) AC = DF; adeoque ipſa triangula æqualia, & ſimilia. Ergo CB = FE, ac propterea BE || CF || AD.

Definitiones.

17. 1. Si Conus ABC plano ADE utcunque per verticem ſectus, rurſus plano ſecetur plano ADE parallelo; Erit facta in illius ſuperficie ſectio FGH HYPERBOLA; cujus planum productum, ſuperficiei oppoſitæ occurrens, faciet ejuſdem nominis ſectionem fgh; Hæ vero binæ conjunctim *Sectiones oppoſitæ* vocantur.

18. 2. Si per Coni verticem A, extraque illius ſuperficiem, (hoc eſt illam nec ſecans nec tangens) tranſeat utcunque planum DAE; Seceturque iterum conus

plano, plano D A E parallelo, Facta in illius ſuperficie ſectio F H G ELLIPSIS dicitur.

3. Quod ſi planum A D E coni ſuperficiem contingat, ſeceturque conus plano, plano A D E parallelo, Facta in illius ſuperficie ſectio F H G dicitur PARABOLA. 19.

Coroll. 1. Hyperbola, & Parabola (cum conum in circuitu non ſecent) ſpatium non claudunt, & continuato Cono, ſimul in infinitum continuantur. Ellipſis vero Conum ambit, & in ſe revoluta ſpatium undique claudit.

Coroll. 2. Liquet Circuli circumferentiam Ellipſibus eſſe annumerandam. Nam ſi planum D A E per verticem ſit plano baſis, vel ſectionis ſubcontrariæ parallelum, Planum huic parallelum faciet in ſuperficie conica Circuli circumferentiam. Ideoque ſub nomine vel ſectionis conicæ in genere, vel Ellipſeos in ſpecie, etiam Circuli circumferentiam comprehenſam intelligat lector. Quod ſemel monitum ſufficiat.

Coroll. 3. Recta conjungens bina quæcunque puncta in ſectione Conicâ, tota cadit intra ſectionem, productaque utrinque extra eandem cadit, neque ei aut ſectioni oppoſitæ amplius occurrit. Patet per prop. 3.

Coroll. 4. Conjungens bina puncta, in oppoſitis ſectionibus ſumpta, cadit extra ſectiones; productaque utrinque intra utramque deinceps continetur, Neutrique earum amplius occurrit. Patet per prop. 3.

Coroll. 5. Unde Recta linea ſectioni conicæ, vel ſect. opp. in pluribus quam duobus punctis non occurrit.

Prop. XI. Theor. X.

Communis interſectio I L *plani alicujus* A M N, *ſuperficiem conicam tangentis, cum plano ſectionis conicæ* F I G, *in unico puncto* I *ſectioni occurrit, totaque extra ſectionem cadit. Hujuſmodi autem occurſus* Contactus *dicitur.* 20.

Nam planum A M N niſi in rectâ A I M ſuperficiei

Conicæ non occurrit ; Unde recta I L, quæ eſt in plano A M N, niſi in puncto I, ubi rectam A I M interſecat, ſectioni non occurrit; Planum vero A I M cadit extra ſuperficiem conicam, proindeque & recta I L extra ſectionem.

20. *Coroll.* 1. Præter I L alia recta in puncto I ſectionem non contingit. Nam erit hujuſmodi recta neceſſario in alio plano per A M, ab A M N diverſo; quod (per *Coroll* 3. prop. 4.) Conum non contingit, i. e. ſecat, proindeque hæc recta ſectionem ſecabit.

21. 22. *Coroll.* 2. Duæ quælibet I L, M N Hyperbolam vel parabolam contingentes, neceſſario concurrunt. Nam (iiſdem manentibus quæ in definitionibus) ſi dicantur parallelæ, erit planorum A I L, A M N contingentes formantium interſectio A B (per *Lemma* 1.) ipſis I L, M N parallela, ideoque in ipſo plano A D E, quod ſectionis plano eſt parallelum. Sed in hyperbola (per *Coroll.* prop. 5.) aliud adhuc planum per A B (nempe ex altera parte plani A D E) poteſt conum contingere; Et in Parabola ipſum A D E conum contingit; Tria igitur plana per unam eandemque rectam A B Conum contingunt, quod (per *Coroll.* prop. 5.) fieri nequit.

23. *Coroll.* 3. In Ellipſi vero, & oppoſitis ſectionibus,
24. Contingenti cuivis I L, erit alia aliqua contingens *i l* parallela. Sit A B recta ſecundum quam planum A I L ſecat planum D A E; ſi per A B agatur aliud planum (per prop. 5.) tangens conicas ſuperficies, formabit hoc contingentem *i l*; Eſtque (ob plana parallela) A B || I L, unde (per *Lem.* 1.) I L || *i l*.

Coroll. 4. Contingens Hyperbolam I L quantumvis producta ſectioni oppoſitæ non occurrit. Nam planum A I L in quo recta I L ſita eſt, non occurrit ſuperficiebus niſi in A I; nec recta I L rectæ A I niſi in I.

Coroll. 5. Rectæ parallelæ duabus plures, Ellipſin vel Sectiones opp. non contingunt. Secus plana per eandem rectam per verticem, duobus plura, conum contingerent.

Prop.

Prop. XII. Theor. XI.

Iiſdem poſitis quæ in definitionibus præcedentibus; In 25.
utrâvis ſectionum oppoſitarum, recta quævis I K
facta à plano per utramvis A D, *vel* A E, *& punctum*
quodvis I *in ſectione, ducto, planumque ſectionis in*
I K *ſecante, in unico hoc puncto* I *ſectioni occurrit; &*
ex unâ parte tota intra ſectionem continetur; ex alterâ
vero manet tota extra utramque ſectionum oppoſitarum.
Similiter, in Parabolâ, recta I K *facta à* 26.
plano per A E *& punctum quodvis* I *in ſectione*
ducto, ſectioni in unico hoc puncto I *occurrit; manetque*
ex unâ parte intra, ex altera extra ſectionem.
2. *At præter iſtiuſmodi rectas* I K, *Quævis alia* I L *per* 25.
punctum quodvis I *ſectionis, in plano ſectionis, ducta,* 26.
Sectioni, vel ſectionibus oppoſitis in binis punctis I, L
occurrit, vel ſaltem ſectionem contingit.

Ob plana parallela erit I K || A D vel A E; unde patet pars 1 per prop. *6*.

2. Cum (ob plana parallela) rectæ omnes I K ſint rectæ A E, vel A D, & ſibi invicem parallelæ; recta I L non erit ipſi A D, vel A E parallela; Proindeque planum per I L & coni verticem non ſecabit planum A D E in A D vel A E, ſed aliâs in A B, quæ in hyperbolâ quidem poteſt cadere vel intra Ang. D A B, id eſt intra ſuperficies Conicas; quo in caſu (per prop. 7.) recta I L huic parallela, occurret ſuperficiei oppoſitæ, hoc eſt, occurret Sectioni oppoſitæ: Vel extra; quo in caſu (per eandem prop.) occurret denuo eidem ſectioni, vel erit in plano ſuperficiem contingente, i. e. Sectionem continget.

In Parabolâ erit A B ſemper extra ſuperficies conicas; Unde I L (per prop. *7*.) vel ſectioni denuo occurret, vel eandem continget.

In Ellipſi, manifeſtum eſt (ex prop. 7.) I L per punctum 27.
quodvis in ſectione I ductam eidem denuo oc-

currere, vel ſaltem ſectionem in eodem puncto I contingere.

28. *Coroll.* Hinc ſi per punctum quodvis B in ſectione
29. conicâ vel utrâvis ſectionum oppoſitarum, ducatur recta
30. BC, quæ ſit rectæ cuivis I L quæ ſectioni vel ſectio-
31. nibus oppoſitis in duobus punctis occurrit, vel quæ
Supple unum caſum ſectionum oppoſitarum. ſectionem, vel alterutram ſectionum oppoſitarum contingit, parallela; Eadem vel ſectioni denuo, vel ſectioni oppoſitæ occurret in C, vel ſaltem ſectionem continget. Nam eo ipſo, quod parallela ſit rectæ I L, liquet non eſſe ex rectarum I K numero; eademque erit ipſius, atque I L ratio.

Prop. XIII. Theor. XII.

32. *Poſitis iis quæ in Hyperbolæ & oppoſitarum ſectionum definitione; Si duo plana* A D I, A E K *ſuperficies conicas ſecundum rectas* D A d, E A e *tangentia, planum ſectionum productum ſecent in rectis* I M i, K M k; *Dico rectas* I M I, K M k *quantumvis productas neutri ſectionum oppoſitarum occurrere.*

Nam ſuperficies conicæ, (in quibus ſectiones ſitæ ſunt) & plana tangentia (in quibus ſunt rectæ I M i K M k) ſibi mutuo non occurrunt niſi in rectis D A d, E A e, quæ (ob plana parallela) ſunt extra planum ſectionum; Igitur rectæ I M i, K M k ſectionibus non occurrunt.

Def. Petito ex re nomine, hujuſmodi rectæ vocantur *Aſymptoti.*

~~32.~~ 33. *Coroll.* 1. Omnis recta A B in plano ſectionum oppoſitarum utrivis aſymptoto parallela, uni ſectionum oppoſitarum idque in unico puncto B occurrit, manetque ex una parte intra, ex altera extra ſectionem. Nam in cono hujuſmodi recta utrivis I M, vel K M parallela, erit (ob plana parallela) etiam rectæ D A vel E A parallela; Unde liquet per prop. 6.

~~32.~~ *Coroll.* 2. Omnis recta M C per aſymptotôn occur-

ſum M intra angulum I M K ducta, Atque omnis recta 33.
D E, quæ ad plagas ſectionum oppoſitarum ſecat utramque aſymptoton, producta utrique ſectionum oppoſitarum occurret. Nam in cono erunt neceſſario parallelæ rectis aliquibus in plano D A E, intra Ang. D A E cadentibus, unde patet per prop. 7.

Coroll. 3. Recta O H N hyperbolam utcunque in H 34.
contingens producta utrique aſymptoto occurret in O, N extra earum occurſum M, & ad eaſdem partes cum ſectione. Nam ſi dicatur tranſire per M, vel eas ad plagas oppoſitas ſecare, vel earum alteri parallelam eſſe, per jam oſtenſa ſecabit ſectionem; Reſtatque ſolummodo caſus propoſitus.

Coroll. 4. Unde quæ binæ O H N, A B C eandem 35.
hyperbolam contingunt, concurrunt neceſſario in D, intra angulum O M N aſymptotis contentum. Quæ vero O H N, a b c oppoſitas ſectiones contingunt, productæ concurrunt intra Angulum a M N, vel c M O qui eſt angulo O M N deinceps; Vel ſaltem ſunt inter ſe parallelæ.

Coroll. 5. Recta B C conjungens bina puncta in hy- 36.
perbola B, C, utrique aſymptoto occurrit ad eaſdem partes cum ſectione; Nam ſi earum uni parallela ſit, occurret ſectioni in unico puncto (per *Coroll.* 1.) contra hypotheſin. Sin utrique occurrat, ſed ad partes oppoſitas, ſectioni oppoſitæ occurret (per *Coroll.* 2.) unde ſectionibus oppoſitis in tribus punctis occurret quod (per *Coroll.* 5. ad *def.* præced.) fieri nequit.

Coroll. 6. Hyperbola & aſymptoti quantumvis pro- 37.
ductæ non concurrunt, propius tamen ad ſe invicem accedunt quam pro dato quovis intervallo. Dato, ab utravis aſymptoto M A, intervallo A B, ductâque B D eidem aſymptoto parallelâ, ſecabit hæc (per *Coroll.* 1.) Sectionem alicubi in C; Unde liquet propoſitum.

Coroll. 7. Ex jam oſtenſis hoc quoque manifeſtum eſt, 34.
Pro vario ſitu puncti H (in *Coroll.* 3.) reſpectu puncti M, punctorum N, O alterum ad punctum M accedere recedente altero, prout ad has vel illas partes punctum

ctum H abierit; At vero eorum neutrum prius puncto M coincidere, quam punctum contactûs H ad diſtantiam infinitam migraverit; Quo in caſu contingens O H N in aſymptoton degenerat; Vel quod idem eſt, Aſymptotos ad punctum ab M infinite diſtans ſectionem contingit.

Prop. XIV. Theor. XIII.

38. *Iiſdem poſitis, quæ in præcedentibus; ſit* I F G K *plani hyperbolæ, vel ſectionum oppoſitarum, interſectio cum plano baſis, occurrens aſymptotis in* I, K, *ſectioni vero in* F, G; *Si, per punctum quodvis in utrâvis ſectionum oppoſitarum* H, *agatur recta* O H N, *rectæ* I K *parallela, aſymptotis occurrens in* O, N, *& ſectioni denuò in* R, *vel eandem fortè in* H *contingens*; *Dico* $IF \times FK = KG \times GI = OH \times HN = NR \times RO$.

Planum baſis in quo eſt recta I K, planumque per O N huic parallelum, facient in ſuperficie conicâ circulorum circumferentias D F G E, P H R Q, quas eorundem planorum interſectiones cum planis, aſymptotos formantibus, *viz.* D I, E K, P O, Q N contingent in D, E, P, Q; & ob plana parallela erit $DI = PO$ & $EK = QN$; Si contingentes coeant in L, l, erit (ob circulum) $LE = LD$, $lQ = lP$, unde (ob ſim. triang.) $LI = LK$, & $lO = lN$, i e. $EK = DI =$ (prius) $PO = QN$; erit ergo $EKq =$ (ob circulum) $KG \times KF = DIq = IF \times IG = POq = OH \times OR = QNq = NR \times NH$: Cum ſit autem

$$\left.\begin{matrix} IF \times IG \\ IF \times IK - IF \times GK \end{matrix}\right\} = \left\{\begin{matrix} GK \times KF \\ GK \times IK - IF \times GK \end{matrix}\right.$$

erit $IF \times IK = GK \times IK$, i e. $IF = GK$; Parique modo erit $OH = RN$; Unde $IF \times IG = IF \times FK = KG \times GI = OH \times OR = OH \times HN = NR \times RO$.

Si contingentes ſint parallelæ, erit (ob parallelas rectas) $DI = EK = QN = PO$. eodemque prorſus modo procedit demonſtratio.

Si OHN contingat hyperbolam, continget & circulum; Unde $PO = OH = QN = NH$, & $POq = OHq = QNq = NHq = OH \times NH = DIq =$ (ut prius) $IF \times FK = KG \times GI$.

Scholium. Oſtendi poſſet $IF = GK$, ductâ per L diametro LST, vel (ſi contingentes ſint parallelæ) diametro his parallelâ, biſecante DE in S, proindeque IK & FG in T; unde $IF = GK$; parique modo $OH = RN$; vel (in caſu contactûs) $OH = HN$. Et, quoad cætera, demonſtratio eſſet eadem.

Prop. XV. Theor. XIV.

In Hyperbola vel ſectionibus oppoſitis, ſi binæ quælibet 39.
rectæ inter ſe parallelæ, & vel utraque ad eandem 40.
Sectionem, vel utraque ad oppoſitas, vel ſingulæ ad ſingulas, utrinque in punctis B, C, F, G, *terminatæ; vel quarum una vel utraque ſectionem, vel ſectiones oppoſitas contingit ut in* C, G, *ſi opus productæ, utrique aſymptoto occurrant in* A, D, E, H; *Dico rectangula* $AB \times BD$, $EF \times FH$, $CD \times CA$, $GH \times GE$ *eſſe ſibi invicem æqualia.*

Si hæ rectæ parallelæ ſint interſectioni plani ſectionis cum plano baſis, hæc propoſitio non differt à præcedenti; Sin minus, per punctorum B, C, G, F bina quælibet ex. gr. C, G agantur ad aſymptotos uſque rectæ ICK, LGQ, quæ interſectioni plani ſectionis cum plano baſis parallelæ intelligantur; Erunt itaque (ob rectas parallelas) triangula DCK, HGQ ſimilia; uti etiam ICA, LGE Unde

$IC : CA :: LG : GE$ &

$CK : CD :: GQ : GH$ ductiſque in ſe ordinatim antecedentibus & conſequentibus, erit $IC \times CK : CA \times CD :: LG \times GQ : GE \times GH$. ſed $IC \times CK =$ (per Prop. præced.) $LG \times GQ$; Ergo $CA \times CD = GE \times GH$. Pari omnino modo (Actis per reliqua puncta B, F rectis, ipſis ICK, LGQ parallelis) demonſtrabitur $AB \times BD = CD \times CA = EF \times FH$, &c.

Coroll. 1. Hinc $CD = BA$, nam

$AB \times BD$ 39.

39. 40. $\left.\begin{array}{r}AB\times BD\\ AB\times BC+AB\times CD\end{array}\right\}=\left\{\begin{array}{l}CD\times CA\\ CD\times BC+DC\times AB,\end{array}\right.$ demptoque communi, erit $AB\times BC=CD\times BC$, *i. e.* $AB=CD$. Eodem modo $EF=GH$.

39. 40. *Coroll.* 2. Unde $AB\times AC=AB\times BD=EF\times FH=EF\times EG$. &c.

41. *Coroll.* 3. Recta A C D ſectionem in C contingens, & ad aſymptotos in A,D terminata à contactu C biſecatur.

Nam A B ubique $=CD$; & in hoc caſu coincidunt
42. puncta B, C. Similiter ſi B C ad ſectiones oppoſitas terminata per aſymptotôn occurſum tranſeat, ibidem biſecabitur. Nam CD ubique $=AB$, *i. e.* $CA=DB$; & in hoc caſu coincidunt A, D.

41. *Coroll.* 4. Unde in caſibus Corollarii 3^{ii} erit $EF\times FH$
42. $=ACq=BDq$ &c.

43. *Coroll.* 5. Coroll. 3^{m}. valet converſim. Nempe ſi recta A C D ad aſymptotos in A, D terminata & ſectioni occurrens in C ibidem biſecetur, eadem ſectionem in C continget. Nam ſi ſectioni denuo occurrat in B, erit
42. $AC=BD=CD$, hoc eſt, punctum B non erit à puncto C diverſum. Similitèr ſi C B ad aſymptoton in D biſecetur, Erit D communis aſymptotôn occurſus.

44. *Coroll.* 6. Binæ rectæ ſectiones oppoſitas contingentes inter ſe parallelæ & ad aſymptotos terminatæ d c a, D C A, ſunt æquales. Nam $dc\times ca=DC\times CA$ & $dc=ca$, $DC=CA$.

44. *Coroll.* 7. Conjungens tactus contingentium parallelarum C *c* per aſymptotôn occurſum M tranſit. Nam per M ductâ C M & productâ; Hæc, ob A D in C biſectam, biſecabit huic parallelam *a d*, hoc eſt, tranſibit per *c*, unde non erit à C *c* diverſa.

Lemma 2.

45. *In rectâ* A D *factâ utcunque* $AB=CD$, *ſumptoque in*
46. *eadem quovis puncto* E; *Si* E *cadat inter* B, *&*
47. C, *dico*

$$BE\times EC=AE\times ED-AC\times CD;\ \textit{vel}$$

$$BE \times EC + AC \times CD = AE \times ED.$$ *vel &c.*

2. *Si* E *cadat inter* C, *&* D, *vel* (*quod eódem redit*) *inter* A, *&* B, *erit*

$$BE \times EC = AC \times CD - AE \times ED;$$ *vel*

$$BE \times EC + AE \times ED = AC \times CD.$$ *&c.*

3. *Si* E *ſit in* AD *productâ, erit*

$$BE \times EC = AE \times ED + AC \times CD;$$ *vel*

$$BE \times EC - AE \times ED = AC \times CD.$$ *&c.*

Nam in primo caſu

$$AE \times ED = AB \times EC + AB \times CD + BE \times EC + BE \times CD \qquad 45.$$

$$AC \times CD = AB \times CD + BE \times CD + \left\{\begin{array}{l} EC \times CD \\ EC \times AB \end{array}\right.$$

Unde $AE \times ED - AC \times CD = BE \times EC$.

In ſecundo caſu,

$$AC \times CD = AB \times ED + BC \times ED + \left\{\begin{array}{l} AB \times CE \\ CD \times CE \end{array}\right\} + BC \times CE, \qquad 46.$$

$$AE \times ED = AB \times ED + BC \times ED + \left\{\begin{array}{l} CE \times ED \\ CE \times CD - CEq; \end{array}\right.$$

Unde

$$AC \times CD - AE \times ED = BC \times CE + CEq = BE \times EC.$$

In tertio caſu,

$$BE \times EC = \left\{\begin{array}{l} BC \times DE + \left\{\begin{array}{l} CD \times DE \\ AB \times DE \end{array}\right\} + DEq + CD \times DE \\ AE \times ED \end{array}\right\} \qquad 47.$$

$$+ \left\{\begin{array}{l} \left\{\begin{array}{l} CDq \\ AB \times CD \end{array}\right\} + BC \times CD \\ AC \times CD. \end{array}\right\}$$

Schol. Coincidentibus punctis B, C, in primo caſu coincidit his punctum E, & evaneſcit $BE \times EC$.

Coincidentibus vero B, C, tranſit caſus ſecundus in illum 5, El. 2. tertius in 6, El. 2. ut cuivis obvium. 48. 49.

Prop. XVI. Theor. XV.

50. 51. *In Hyperbola & ſectionibus oppoſitis, ſi binæ quælibet*
52. 53. *rectæ* KN, ST, *vel utraque ad eandem ſectionem,*
54. 55. *vel utraque ad oppoſitas, vel ſingulæ ad ſingulas, vel*
56. *altera ad eandem, altera ad oppoſitas in* K, N; S, T, *utrinque terminatæ, & ſi opus productæ, ſibi mutuo occurrant in* E, *& utrique aſymptoto in* R, V, M, L; *Dico*

$$\left.\begin{array}{l} KM\times MN \\ LN\times MN \end{array}\right\} : RT\times TV :: KE\times EN : SE\times ET.$$

57. 58. 2. *Idem erit de quadratis hujuſmodi rectarum, quoties*
59. 60. *harum una vel utraque ſectionem, vel utramque ſectionum oppoſitarum, contingit, vel harum una per aſymptotôn occurſum tranſit.*

50. 51. Per utriuſvis rectarum NK utrumlibet occurſum
52. &c. cum ſectione N, agatur recta XNY, rectæ alteri ST parallela, aſymptotis occurrens in X, Y; erunt triangula LNY, LER ſimilia, uti etiam MNX, MEV; Unde

$$LE : LN :: ER : NY$$
$$ME : MN :: EV : NX.$$ Ductiſque &c.

$LE\times ME : LN\times MN :: ER\times EV : NY\times NX$ id eſt (per lemma præced.) pro vario ſitu puncti E, & per prop. 15. cum *Coroll.* 2.

$$\left.\begin{array}{l} KE\times EN+KM\times MN \\ KM\times MN-KE\times EN \\ KE\times EN--KM\times MN \end{array}\right\} : \left\{\begin{array}{c} LN\times MN \\ \text{id eſt} \\ KM\times MN \end{array}\right\} :: \left\{\begin{array}{l} SE\times ET+RT\times TV \\ RT\times TV-SE\times ET \\ SE\times ET-RT\times TV \end{array}\right\} : \left\{\begin{array}{c} NY\times NX \\ \text{id eſt} \\ RT\times TV \end{array}\right.$$

Unde in primo & ſecundo caſu div. in tertio comp. $KE\times EN : KM\times MN :: SE\times ET : RT\times TV$, unde alternando liquet propoſitum.

2. Oſtenſum eſt in parte prima $KM\times MN : RT\times TV :: KE\times EN : SE\times ET$: unde coeuntibus punctis K & N, T & S vel M & L &c. erit

Fig. 57. 58. $KMq : RT\times TV :: KEq : SE\times ET.$

Fig. 60. $KMq : RT\times TV :: KE\times EN : SE\times ET.$

Fig. 59. $KMq : RTq :: KEq : TEq.$ Et ſic in cæteris caſibus.

Scholium. Coeuntibus punctis K, N; S, T &c. Lemma præcedens in hac demonſtratione adhibitum non differt à 5, vel 6, El. 2, ut ſupra adnotavimus.

Coroll. Si angulus aſymptotôn rectus intelligatur, & 56.
rectarum una KN, parallela ſit, vel coincidat rectæ
alicui ACB per aſymptotôn occurſum C tranſeunti,
altera vero ST parallela rectæ DB in occurſu rectæ
AB unam ſectionum oppoſitarum contingenti; Erit
$KE \times EN = SE \times ET$. Nam ob ang. rect. & $DB =$
BI, erit per 31. El. 3^{tii}. $CBq = DBq$, & (per *Coroll.* 4.
prop. 15.) $RT \times TV = DBq$, & $KM \times MN = CBq$.
& ſic in cæteris caſibus.

Prop. XVII. Theor. XVI.

In Hyperbola & Sectionibus oppoſitis ſi Recta quævis 61. 62.
AB *ad eandem, vel ad oppoſitas ſectiones utrinque* 63. 64.
terminata, & ſi opus producta, Vel binæ quælibet 65.
rectæ AB, CD *inter ſe parallelæ, & vel utraque ad* *Supple figuras reliquorum caſuum.*
eandem ſectionem, vel utraque ad oppoſitas, vel ſingulæ ad ſingulas utrinque terminatæ, & ſi opus productæ, Binis quibuſcunque FG, IH *inter ſe parallelis, & vel utriſque ad eandem ſectionem, vel utriſque ad oppoſitas, vel ſingulis ad ſingulas utrinque terminatis, & ſi opus productis utcunque occurrant, ut in* E, T, *vel* E, T, S, V *punctis; Erunt rectangula ex ſegmentis ejuſdem rectæ, vel binarum inter ſe parallelarum, proportionalia rectangulis ex conterminis ſegmentis binarum reliquarum, hoc eſt*

$AE \times EB : GE \times EF :: AT \times TB : IT \times TH$, &
$AE \times EB : GE \times EF :: CV \times VD : HV \times VI$. *&*
ſic de binis quibuſvis punctis.

2. *Sin recta quævis* AET *utrivis aſymptoto parallela,* 66. 67.
ſectioni in unico puncto A *occurrens, binis iſtiuſmodi* *Supple figuras reliquorum caſuum.*
parallelis IHT, FGE *occurrat in* T, E; *erit*

$HT \times TI : GE \times EF :: TA : EA$.

Eadem de rectarum contingentium quadratis intellige.

Rectæ omnes (ſi opus productæ) occurrant aſymptotis in O, P, Q, R &c. ut in figuris. Erit ex præcedenti

$$AE \times EB : GE \times EF :: AO \times OB : GK \times KF$$

$$\& \ AT \times TB : IT \times TH :: AO \times OB \left\{ \begin{matrix} HM \times MI \\ GK \times KF \end{matrix} \right\}$$

Unde $AE \times EB : GE \times EF :: AT \times TB : IT \times TH$. Pari modo ob $AO \times OB = CQ \times QD$ & $GK \times KF = HM \times MI$ probabitur

$$AE \times EB : GE \times EF :: CV \times VD : HV \times VI.$$

Nec aliter ratiocinabere de cæteris punctis; & in caſibus contingentium.

66. 67. *Supple caſus omiſſos.* 2. Sit aſymptotôn occurſus B, & connectatur BA, Per puncta T, E ipſi BA parallelæ, agantur DTS, KEN; quæ occurrent neceſſariò utrique aſymptoto in D, S; K, N, adeoque (per Coroll. 2. prop. 13.) utrique ſectionum oppoſitarum in O, P; L, M; ſint vero K, D puncta in aſymptoto ipſi AET parallela.

Erit (ob parallelas rectas) $AB = DT = KE$ & $BK = AE$, $BD = AT$.

Porro (ex priore parte, & ex lemmate 2.) erit in caſu Figuræ 66.

$$HT \times TI : GE \times EF :: \left\{ \begin{matrix} OT \times TP \\ DT \times TS + OS \times SP \\ AB \times TS + ABq \end{matrix} \right\} : \left\{ \begin{matrix} LE \times EM \\ KE \times EN + LN \times NM \\ AB \times EN + ABq \end{matrix} \right\}$$

$$:: \left\{ \begin{matrix} TS + AB \\ DS \end{matrix} \right\} : \left\{ \begin{matrix} EN + AB \\ NK \end{matrix} \right\} :: (\text{ob ſim.triang.}) \left\{ \begin{matrix} DB \\ TA \end{matrix} \right\} : \left\{ \begin{matrix} KB \\ EA \end{matrix} \right\}$$

Nec aliter in caſu contactûs.

Eodem modo, per Lemma 2. (pro vario ſitu punctorum T, E, mutatis mutandis) patebit in omnibus caſibus.

Prop. XVIII. Theor. XVII.

68. 69. 70. 71. 72. *Supple reliquos caſus.* *Quæ in Hyperbola, & ſectionibus oppoſitis ſuperiori propoſitione oſtendimus, propoſitum ſit in Ellipſi & Parabola oſtendere. Scilicet, (poſito rectas* RO, LM, *parallelas, ſectionique in* R, O, L, M, *occurrentes, rectis* HF, YZ *parallelis, & ſectioni in* H, F, Y, Z *occurrentibus,*

occurrentibus, in punctis G, E, X, V, *occurrere,) erit*

$$GO \times GR : LE \times EM :: HG \times GF : HE \times EF$$

$$GO \times GR : HG \times GF :: LX \times XM : ZX \times XY$$

& ſic de binis quibuſvis punctis.

2. *Sin Recta* F H *occurrens Parabolæ in unico puncto* F, *occurrat parallelis* R G O, L E M, *in* G, E; *erit* 75. *Supple reliquos caſus.*

$$FG : FE :: GO \times GR : LE \times EM.$$

Eadem de contingentium quadratis intellige.

In Ellipſi, vel Parabola in Conica ſuperficie factâ, ſint rectæ eædem quæ in figuris 68, 69. &c. neglectâ tantum in præſens rectâ Z Y. 73. 74. *Supple caſus reliquos.*

Per rectarum R O, L M alterutram R O, & coni verticem I transſeat planum R I O, ſecans ſuperficies conicas in rectis I R, I O; Secundum quas plana I R K, I O N ſuperficies contingant, ſe mutuo ſecantia in I T, & planum ſectionis in R K, O N.

Per L M tranſeat planum, plano R I O parallelum, faciens ſectiones oppoſitas L D M A S, ſecanſque plana tangentia in rectis T K, T N quæ erunt propterea aſymptoti.

Per H F & coni verticem I tranſeat planum, ſecans ſuperficies conicas in rectis H I A, I D F, planum R I O in I G, planum ſectionum in A B C D E, planaque aſymptotos formantia in I B, I C.

In fig. 74. rectam IC, vitandæ confuſionis ergô non expreſſimus.

Recta A D eſt ad alteram, ſive utramque ſectionum oppoſitarum in A, D terminata, occurritque aſymptotis in B, C.

Per punctum D, in plano ſectionum oppoſitarum agatur, uſque ad aſymptotos, recta P D Q parallela K L E M N.

Ob plana parallela, & rectas parallelas P D Q, K L E M N, ſimilia erunt triangula R G I, & P D B; G I O, & D C Q. Pariter H I G & H A E; F D E, & F I G, Unde

$$IG : DE :: GF : FE \ \&$$

$$IG : AE :: HG :: HE \quad \text{Ductiſque \&c.}$$

$$IGq : AE \times DE :: HG \times GF : HE \times FE. \quad \text{Rurſum}$$

IG : GO :: CD : DQ
IG : GR :: BD : DP Ductiſque &c.

$IGq : GO \times GR :: CD \times BD : \left\{\begin{matrix} DQ \times DP \\ KM \times MN \end{matrix}\right\}$

:: (per prop. 16.) AE × ED : LE × EM & altern.
IGq : AE × ED :: GO × GR : LE × EM :: (prius) HG × GF : HE × FE.

Probandum reſtat

68, 69. RG × GO : HG × GF :: LX × XM : ZX × XY
70, 71. Jam vero oſtendimus
72. RG × GO : HG × GF :: LE × EM : HE × EF
Supple reliquos caſus. & pari ratione erit
LE × EM : HE × EF :: LX × XM : ZX × XY
unde
RG × GO : HG × GF :: LX × XM : ZX × XY

Pariter de punctis E, V. Nec aliter in rectis contingentibus

75. 2. Factis ut in priore parte. Cum recta FH re-
& Fig. 26. ſupra. ſpondeat rectæ IK in fig. prop. 12, pro Parabola, recta
IA quæ fit à plano per rectam FH & verticem I re-
Supple reliquos caſus. ſpondebit rectæ AD in eâdem figura; hoc eſt, erit
FH parallela IA unde IG = AE.

Eſtque ob ſim. triang.
FG : FE :: IG : DE :: IGq : IG × DE = AE × DE
oſtendetur verò ut ſupra
IGq : AE × DE :: GO × GR : LE × EM unde
FG : FE :: GO × GR : LE × EM. Pariterque in caſibus contactûs.

Scholium. Theorema hoc cum præcedenti licuiſſet ſimul unâ propoſitione complecti, eademque operâ è cono demonſtraſſe: Cum tamen in hyperbola & ſectionibus opp. hæc proprietas etiam in plano ſe proderet, quo Tyronibus aliquatenus conſultum eſſet, libuit ſeorſim demonſtrare.

SECTIO II.

Prop. XIX. Theor. XVIII.

In omni Sectione conica, & Sectionibus oppoſitis, ſi binæ rectæ BI, CI *contingentes ſectionem, vel ſectiones oppoſitas in extremis rectæ cujuſvis* BC, *ſectioni, vel ſect. opp. in binis punctis* B, C *occurrentis, concurrant in* I, (*vel in Ellipſi & ſect. opp. ſint forte inter ſe parallelæ*) *& ex utrâvis parte tactus conjungentis* BC *ducatur recta* EF, *ipſi* BC *parallela, quæ* (*ſi opus producta*) *occurrat ſectioni, vel ſectioni oppoſitæ, vel utrique ſectionum oppoſitarum in binis punctis* E, F, *& rectis contingentibus in* D, G; *Dico* DE = FG.

76. 77. 78.

Tam Parabolæ quam Ellipſeos figura numero 76 (per errorem) deſignatur.

Rectæ OHN, QKR *ſpectant ad prop. ſeq.*

FActâ in ſuperficie conica, ſectione, vel ſectionibus oppoſitis, poſitiſque ut in prioribus figuris. Per coni verticem A ducantur AB, AC; Plana per rectas AB, BI & AC, CI ſuperficiem contingent.

79.

Pro Hyperb. Parab. & Ellipſi.

Supple figuram ſect. opp.

Per DEFG tranſeat planum, plano ABC parallelum, faciens ſectionem vel ſectiones EKF, & ſecans plana tangentia in DH, HG; Erit EKF hyperbola, vel ſectiones oppoſitæ; eruntque HD, HG aſymptoti, ac proinde DE = FG.

In Hyperbola & ſectionibus oppoſitis, hæc propoſitio in plano demonſtrari poteſt in hunc modum.

Sint AV, AT aſymptoti, occurrentes contingentibus (ſi opus productis) in T, V, rectæ BC (ſi opus productæ) in X, Y, & rectæ DG in R, S; biſectâque BC in M, juncta IM [vel ∥ BI, CI] ſecet EF in L. 77. 78.

Per prop. 16, IBq : ICq :: BTq : VCq; ergo BC & TV parallelæ ſunt, unde (ob BM = MC) recta IM biſecabit connexam TV in Z.

Rurſus (ob BX = YC & BM = MC) erit XM =

$= MY$; ſi ergo per aſymptotôn occurſum A ducatur recta AM, hæc (propter parallelas XY, TV & biſectas in M, Z) tranſibit per Z, hoc eſt non differet ab IM primo ductâ.

Jamque (ob $BM = MC$,) erit $DL = LG$; & (ob $XM = MY$) erit $RL = LS$; unde $DR = SG$; ſed & $RE = SF$, unde $DE = GF$.

Coroll. 1. $DF = GE$.

Coroll. 2. Si (coeuntibus punctis E, F,) recta DG ſectionem contingat, à tactu biſecabitur. Si vero puncta E, F ſint ad ſectiones oppoſitas, recta DG in contingentem abire non poteſt, ut ex ſupra dictis manifeſtum.

Prop. XX. Theor. XIX.

76. 77. 78. *Supple figuras contingentium parallelarum.*

Iiſdem poſitis, Si per contingentium occurſum I, *& medium punctum* M *tactus conjungentis BC, ducatur recta* IM; *vel ſi contingentes ſint parallelæ, per* M *ducatur* IM *ipſis contingentibus parallela, Hæc rectas omnes* EF *ipſi* BC *parallelas, & ad ſectionem vel ſectiones oppoſitas utrinque terminatas, bifariam ſecabit.*

Nam ob ſimilia triang. IBC, IDG & $BM = MC$, erit etiam $DL = LG$; eſtque (per præced.) $DE = FG$, & $DF = GE$, unde additis vel demptis æqualibus erit $LE = LF$.

Si contingentes ſint parallelæ, erit $BM = DL = MC = LG$ &c. ut prius,

Def. Rectæ IM hoc modo genitæ, atque infinitè extenſæ *Diametri* appellantur.

Ipſæ vero MC, vel MB, atque omnes his parallelæ LF, vel LE, intra ſectionem, vel inter ſectiones oppoſitas, dicuntur *Ordinatæ* ad diametrum IM.

Coroll. 1. Ob ordinatas ſic biſectas in Hyperbola, & Parabola, (quarum curvæ poſt BC infinite ſe utrinque extendunt) diameter in unico tantum puncto H ſectioni occurrit; In hyperbola tamen occurret, eâdem de cauſa, ſectioni oppoſitæ in K. Ellipſi occurret in binis punctis H, K. Inter ſectiones oppoſitas neutri earum omninò occurrit.

Def. In Hyperbola, vel Sect. opp. & Ellipſi, Diametri infinite extenſæ pars H K *Diameter tranſverſa* dicitur. 76. 77.

Et in omnibus ſectionibus punctum H, vel K *Vertex* dicitur; & in opp. Sect. & Ellipſi dicitur vertex K vertici H *Oppoſitus*, & viciſſim.

In oppoſitis ſectionibus Diametri H K utrique ſectioni occurrentes vocantur Diametri *Determinatæ*; Diametri vero inter ſectiones *Indeterminatæ*. Utriuſque nominis ratio in aperto eſt. 77. 78.

In omnibus ſectionibus partes H M, H L, vel K M, K L vocantur *Diametri Interceptæ* vel *Abſciſſæ*. 76. 77.

Coroll. 2. In Hyperbola & Ellipſi; cum ſit B M × M C = M C q, E L × L F = L F q erit (per prop. 17. & 18.) K M × M H : K L × L H :: M C q : L F q. Et ſic ubique.

Coroll. 3. In Parabola, cum Diametri in unico tantum puncto H ſectioni occurrant, liquet (per prop. 12.) eas non diverſas eſſe à rectis I K factis à plano per contactum A D, planumque ſectionis in I K ſecante; adeoque (ob planum A D E plano ſectionis parallelum,) eſſe omnes inter ſe parallelas; Et porro idem præſtare, quod eædem rectæ I K in prop. 18. 76. & *In fig. prop. 12 pro Parabola.*

Coroll. 4. Et propter ordinatas biſectas, (per prop. 18.) Erit in Parabola 76.

H M : H L : : M C q : L F q; Et ſic ubique.

Coroll. 5. Per *Coroll.* 2. liquet ordinatas ad quamlibet diametrum in Ellipſi, quo magis ab utroque vertice diſtant, eo majores eſſe, donec ad medium diametri punctum deventum ſit; Eritque ordinata per hoc punctum maxima. Et quæ binæ æqualiter ab utroque vertice diſtant ſunt æquales; Et converſim. Ita enim ſe res habet in rectangulis ex ſegmentis diametri, quibus ordinatarum conterminarum quadrata proportionalia ſunt. 76.

76. 77. *Coroll.* 6. Similiter in Hyperbola & Parabola, quo longiùs ordinatæ à diametri vertice diſtant, eo ſunt majores. Et in oppoſitis ſectionibus, quæ binæ æqualiter ab utroque ejus vertice diſtant, ſunt æquales; Et converſim.

76. 77. *Coroll.* 7. In Hyperbola & Parabola, quarum curvæ infinite ſe extendunt, fiunt tandem ordinatæ datâ quâvis rectâ lf majores. Nam ſi in Hyperbola ita ſumatur punctum L ut ſit $KL \times LH : KM \times MH \sqsubset lf\,q : MC\,q$ erit (propter *coroll.* 2.) $LF \sqsubset lf$. Eodem modo (mutatis mutandis) liquet in Parab. per *Coroll.* 4. Unde porrò recta diametro cuivis parallela, ſectioni, vel utrique ſectionum oppoſitarum neceſſariò occurret.

76. 77. *Coroll.* 8. In omni ſectione, Quæ H N, vel K P per verticem H, vel K ducitur ordinatis parallela, Sectionem contingit; Et converſim. Nam ſi aliqua ejus pars ſit intra ſectionem, à diametro biſecabitur, quæ tamen huic non occurrit niſi in ipſa ſectione. 2. Quæ in H vel K contingit, erit ordinatis parallela. Nam binæ rectæ diverſæ in eodem puncto non contingunt.

80. 81. *Coroll.* 9. In Ellipſi & Sectionibus oppoſitis, Recta H K conjungens tactus contingentium parallelarum eſt diameter rectarum B C, E F hiſce contingentibus parallelarum. Nam ſi quævis alia V R dicatur earum diameter, rectæ in ejuſdem extremis V, R, rectis B C, F E parallelæ, (per præced. *Coroll.*) ſectionem contingent, unde aut plures duabus rectis parallelis ſectionem, vel ſect. opp. contingent, contra *Coroll.* 5. prop. 11, aut non erit V R ab H K diverſa.

80. 81. 82. 83. *Coroll.* 10. In omni ſectione conicâ, & in ſect. opp. & inter ſect. opp. Quæ binas rectas inter ſe parallelas, vel utraſque ad eandem ſectionem, vel utraſque ad oppoſitas, vel ſingulas ad ſingulas utrinque terminatas, biſecat, Erit harum, atque omnium his parallelarum diameter. Nam propria diameter has biſecat, & binæ rectæ à ſe invicem diverſæ has biſecare non poſſunt.

80. 81. 82. *Coroll.* 11. Et in omnibus ſectionibus, & in ſect.

opp. ſi recta H O in quovis puncto H ſectionem, vel utramvis ſectionum oppoſitarum contingat, atque huic parallela quævis B C ſectioni, vel utrivis ſectionum oppoſitarum in punctis B, C occurrat, Recta H M, quæ tactum H, & medium punctum M rectæ B C conjungit, erit hujus atque huic parallelarum diameter. Patet ut *Coroll.* 9.

Coroll. 12. In Parabola, Si in puncto quolibet H contingat H O, ſitque huic parallela quælibet B C utrinque ad ſectionem terminata; Quæ H M per tactum H ducitur diametro cuivis S T parallela, erit rectæ B C, atque huic parallelarum, diameter. Nam harum diameter (per *Coroll.* præced.) per H tranſit, & (per *Coroll.* 3.) eſt parallela S T, ideoque non eſt ab H M diverſa. 82.

Coroll. 13. Parabolæ diameter, præter ordinatas ſuas, nullam rectam utrinque ad ſectionem terminatam biſecat. Nam propria diameter hanc biſecat, Et à duabus rectis parallelis, à ſe invicem diverſis, biſecari nequit.

Coroll. 14. Ideoque in Parabola, duæ rectæ ad ſectionem utrinque terminatæ ſe mutuo non biſecant.

Coroll. 15. Contingens parabolam omnibus diametris productis occurrit. Et quælibet recta utrinque ad parabolam terminata & ſi opus producta, omnibus diametris ſi opus productis occurrit. Nam quæ uni parallelarum occurrit, occurrit omnibus. 82.

Coroll. 16. Si in Parabolâ recta quævis H M, diametro cuivis S T parallela, rectam quamvis B C utrinque ad ſectionem terminatam biſecet, Erit hujus atque huic parallelarum diameter. Nam omnes diametri ſunt parallelæ (per *Coroll.* 3.) Et (per 13.) præter ordinatas ſuas nullam omnino rectam ad ſectionem utrinque terminatam biſecant. 82.

Coroll. 17. In omnibus ſectionibus, & ſectionibus oppoſitis, ex diametrorum geneſi liquet Contingentes in extremis rectæ cujuſvis B C, ad diametrum quamlibet H *b* ordinatæ, ſuper eâdem diametro coire in aliquo puncto I; aut eſſe eidem parallelas. 76.77 78.

Prop. XXI. Theor. XX.

84. 85. *In Ellipſi, & ſectionibus oppoſitis, Quæ* DHE *per medium punctum* H *diametri cujuſvis* AB (*quæ in opp. ſect. ſit determinata*) *utcunque ducitur, Sectioni vel ſectionibus oppoſitis in punctis* E, D *occurrens, à puncto* H *biſariam dividitur.*

Si DHE ſit ad diametrum AB ordinata, jam liquet propoſitum. Sin minùs; è punctis E, D intelligantur ordinatæ ad diametrum AB, rectæ EG, DF; Quæ parallelæ cum ſint, erunt triangula HEG, HDF ſimiſimilia;

Per *Coroll.* 2. præced. erit
BF × FA : BG × GA :: DFq : GEq :: (propter ſim. triang.) FHq : GHq unde alternando, & componendo in Ellipſi, dividendo in oppoſitis ſectionibus,

$$\left.\begin{array}{l} BF\times FA + FHq \\ FHq - BF\times FA \\ \quad i.e.\ HAq \end{array}\right\} : FHq :: \left\{\begin{array}{l} BG\times GA + GHq \\ GHq - BG\times GA \\ \quad i.e.\ HAq \end{array}\right\} : GHq$$

Unde FH = GH & ob ſim. triang. inde DH = HE.

Prop. XXII. Theor. XXI.

86. 87. *In Hyperbola, vel oppoſitis ſectionibus, omnes diametri coeunt in occurſu aſymptotôn* H, *quarum determinatæ ibidem ſe mutuò biſecant. Et in Ellipſi omnes diametri coeunt, & ſe mutuò biſecant, in communi quodam intra ſectionem puncto* H.

86. Cum diametri determinatæ in hyperbola vel opp. ſect. jungant tactus contingentium parallelarum, tranſeunt omnes (per *Coroll.* 7. prop. 15.) per punctum H, & (per *Coroll.* 3. ejuſdem prop. 15.) ibidem biſecantur. Indeterminatæ vero biſecant omnes ordinatas ſuas inter ſectiones, quarum una neceſſariò per H tranſit, & ibidem biſecatur; Omnes itaque diametri indeterminatæ pariter per H tranſeunt.

In Ellipſi, ſi per duarum quarumlibet diametrorum A B; E D media puncta C, H ducatur F C H G, ſectioni utrinque occurrens, hæc (per præced.) tam in C, quam in H biſecabitur; Coeunt itaque puncta C, H; unde liquet propoſitum. 87.

Def. Punctum H *Centrum*, dicitur. 86. 87.

Corollaria ad hanc & præcedentes aliquot propoſitiones.

Coroll. 1. Omnis recta E H per punctum quodvis E Ellipſeos, vel utriuſvis ſectionum oppoſitarum, & centrum H ducta, eſt diameter rectarum contingenti in puncto E parallelarum. 88. 89.

Coroll. 2. Et omnis recta H M, per centrum H, & medium punctum M rectæ cujuſvis B C ad Ellipſim, vel hyperbolam, vel ſectiones oppoſitas utrinque terminatæ, ducta, eſt hujus atque huic parallelarum Diameter. 88. 89.

Nam biſecabit ad centrum (per prop. præced.) rectam rectæ B C parallelam; unde (per *Coroll.* 10. prop. 20.) erit harum diameter.

Coroll. 3. In Ellipſi, & Hyperbolâ, & inter ſectiones oppoſitas; Diameter, præter ordinatas ſuas, nullam rectam ad ſectionem, vel ſectiones oppoſitas terminatam (niſi in centro) biſecat. Nam omnis hujuſmodi recta à propria diametro biſecatur, & à binis rectis à ſe invicem diverſis (niſi in communi earum occurſu) biſecari nequit.

Coroll. 4. Atque hinc binæ rectæ ad Ellipſin, Hyperbolam, vel Sectiones oppoſitas, terminatæ (niſi in Centro) ſe mutuò non biſecant.

Coroll. 5. In oppoſitis Sectionibus, & in Ellipſi, partes oppoſitæ à quavis diametro A B abſciſſæ ſuprapoſitæ congruunt. Nam ductâ aliâ quâvis diametro E H D, & ſuprapoſitis figuris, congruent (ob æquales rectas & angulum communem) puncta H, A, D, punctis H, B, E reſpectivè; & ſic de cæteris punctis. 86 87.

Coroll. 6. Unde ipſæ ſectiones oppoſitæ ſupra poſitæ congruent. 86.

Prop. XXIII. Theor. XXII.

90. 91. *In ſectionibus oppoſitis, & in Ellipſi, Si per centrum* C *agatur recta* E D *diametri alicujus* A B (*quæ in opp. Sect. ſit determinata*) *ordinatis* G I *parallela, Biſecabit hæc rectas omnes* F G *diametro* A B *parallelas, & ad ſectionem, vel ſectiones oppoſitas, utrinque in* F, G *terminatas. Et converſim.*

A punctis F, G intelligantur ad diametrum A B ordinatæ F H, G I; Ob rectas parallelas erit $FH = GI$, unde (per converſ. *Coroll.* 5. & 6. prop. 20.) $AH = BI$, & (ob $AC = CB$) $HC = CI =$ (ob rectas parall.) $GM = MF$.

2. Si F G biſecetur à recta E D in M, erit parallela A B. Nam factis ut ſupra, Erit (ob F H parall. G I) $FM : HC :: MG : CI$ *i. e.* (ob $FM =$ ex hyp. MG) $HC = CI$, unde & $AH = BI$, & (per *Coroll.* 5. & 6. prop. 20.) $FH = GI$: Ergo cum ſit $HF \parallel GI$ erit $FG \parallel AB$. 90. 91.

Coroll. 1. Hinc recta F G eſt ad diametrum E D ordinata.

Def. Diameter E D infinitè extenſa dicitur diametro A B *Conjugata*. 91.

90. In Ellipſi eadem utrinque in E, D ad Sectionem terminata, Vel in oppoſitis ſectionibus, abſciſſis utrinque C D, C E, ut ſint eædem contingentis in utrovis vertice B, ad aſymptotos utrinque terminatæ, partibus B K, B L æquales, (proindéque æquales inter ſe) Dicitur D E *Secunda diameter* diametro A B *conjugata*, & viciſſim.

Coroll. 2. Conjugatarum diametrorum ordinatæ ſunt diametris ſuis reciproce parallelæ, Conjugatæ diametri ſunt ad ſe mutuò ordinatæ, Et cujuſvis diametri unica eſt conjugata.

90. *Coroll.* 3. Ordinatarum ad diametrum quamvis E D inter ſectiones oppoſitas (*i. e.* indeterminatam) minima

eſt quæ per centrum tranſit, & à centro remotiores ſunt propioribus majores, fiuntque tandem datâ quâvis rectâ S T majores. Nam quo major eſt C M *i. e.* I G, eo major B I, & proinde eo major C I = M G. Sumptâque C I $\sqsubset$ S T ductâque I G parallelâ E D, & G M parallelâ I C, erit ordinata G M = C I, *i. e.* $\sqsubset$ S T.

90. *Coroll.* 4. In oppoſitis ſectionibus, conjugatarum diametrorum una eſt determinata, altera indeterminata. Nam ordinata ad diametrum determinatam occurrit utrique aſymptoto in P, Q ad eaſdem partes cum ſectione, unde huic parallela extra angulum aſymptotôn cadet. Et vice verſa.

90. *Coroll.* 5. Cum omnis recta per quodvis punctum in utrâvis ſectionum oppoſitarum (modo aſymptoto non parallela, nec ſectioni oppoſitæ occurrens) eidem ſectioni denuò occurrat, ſitque ad diametrum aliquam determinatam ordinata, habeatque rectam aliquam per centrum extra angulum aſymptotôn cadentem ſibi parallelam; Atque omnis recta intra angulum aſymptotôn cadens ſit diameter determinata: Liquet omnes rectas per centrum ductas eſſe diametros; Exceptis tantummodò Aſymptotis, quæ quaſi limites ſunt, qui determinatas diametros ab indeterminatis diſterminant, & in quas diametri (ut de contingentibus jam olim dictum eſt) ultimò degenerant. Nam abeunte puncto B in infinitum, Contingens K B L (accedente puncto L ad C,) cumque hâc huic parallela E D, atque has conjungens C B, aſymptoto C K omnes coincident, ut advertenti facilè patebit.

In Ellipſi omnes omnino rectæ per centrum (per *Coroll.* 1. prop. 22.) erunt diametri.

Coroll. 6. In Hyperbolâ, vel oppoſitis ſectionibus, & Ellipſi, Contingens in vertice cujuſlibet diametri occurrit omnibus diametris, ſi opus productis, exceptâ duntaxat huic conjugatâ. Idem de ordinatis ad quamlibet diametrum (ſi opus productis) intellige.

90. 91. *Coroll.* 7. In Hyperbolâ, & Ellipſi, C B q : C D q :: A I × I B : I G q. Nam in hyperbola, C D q = B L q,

&

& $OI \times IG = IGq$, unde patet per prop. 16. In Ellipſi $CBq = AC \times CB$, unde patet per *Coroll.* 2. prop. 20.

90. *Coroll.* 8. Si angulus aſymptotôn ſit rectus, erit ubique $CB = CD$, ſi acutus $CB \sqsubset CD$, ſi obtuſus $CB \sqsupset CD$. Nam ſemicirculus diametro $KL = ED$ in primo caſu tranſibit per C, in ſecundo citrà, in tertio ultra C, unde CB reſpectivè $=, \sqsubset, \sqsupset BL$ *i. e.* CD.

91. ſi Ellipſis ſit circulus erit ubique $CB = CD$.

Prop. XXIV. Theor. XXIII.

92. 23. *In Hyperbolâ & Ellipſi, ſit quælibet diameter* DF, (*quæ tamen in hyperbolâ ſit determinata*) *cujus vertex* D, *vertex oppoſitus* F, *ſecunda diameter huic conjugata* GB, *& ad* DF *quælibet ordinata* KN; *Fiat ipſis* $FD : GB$: *tertia proportionalis* LR, *in quâ* (*ultra* R *productâ in Hyperbolâ*) *ſumatur punctum* M, *ut ſit* $FD : DK :: LR : MR$; Dico $NKq = FK \times MR = DK \times LM$.

Ob $FD : DK :: LR : MR$ erit componendo in Hyperb. dividendo in Ellipſi

$$\left.\begin{matrix} FD \pm DK \\ FK \end{matrix}\right\} : DK :: \left\{\begin{matrix} LR \pm MR \\ LM \end{matrix}\right\} : MR,$$

Unde $FK \times MR = DK \times LM$. Rurſum ob $FD : GB : LR$ ∺ erit

$FD : LR :: FDq : GBq ::$ (horum ſubquadrupla) $CDq : CBq ::$ (per *Coroll.* 7. præced.) $DK \times FK : NKq ::$ (ex hyp.) $DK : MR :: DK \times FK : MR \times FK$, Ergo $NKq = MR \times FK = DK \times LM$.

Def. Recta LR dicitur *Latus rectum* ſive *Parameter*.

Rectangulum ex diametro quâvis & ſuâ parametro $FD \times LR$ vocatur *Figura* iſtius diametri.

Coroll. 1. In hyperbolâ, ſi angulus aſymptotôn ſit rectus, erit omnis diameter ſuæ parametro æqualis &c. patet ex *Coroll.* 8. præced.

Def. Hujuſmodi vero hyperbola, ob tranſverſam diametrum (quæ & Latus tranſverſum quandoque dicitur) parametro ſive lateri recto æqualem, dicitur Hyperbola *Æquilatera*.

Coroll. 2. Si Ellipſis ſit Circulus, erunt omnes parametri diametris ſuis & ſibi ipſis æquales.

Coroll. 3. Et in quâlibet Ellipſi, binæ quælibet diametri conjugatæ ſunt duæ mediæ proportionales inter earum parametros. Idem in Hyperbolâ intellige.

Coroll. 4. G B q = figuræ diametri F D; & C B q = $\frac{1}{4}$ ejuſdem fig.

Coroll. 5. D F : L R : : D K × F K : N K q. Patet ſupra. 92. 93.

Coroll. 6. Quadratum ordinatæ N K in Hyperbolâ 93.
excedit rectangulum ex parametro L R & diametro interceptâ D K, *i. e.* L R × D K, rectangulo D K × M R, quod ſimile eſt figuræ diametri F D × L R. Nam N K q = D K × L M = L R × D K + M R × D K. Et F D : D K : : L R : M R, *i. e.* F D × L R ſimile D K × M R.

Coroll. 7. In Ellipſi vero, Quadratum ex N K deficit 93.
à rectangulo L R × D K, rectangulo D K × M R, quod ſimile eſt figuræ diametri; Et à rectangulo L R × F K, rectangulo F K × L M, quod eidem figuræ etiam ſimile eſt. Nam N K q = D K × L M = R L × D K — M R × D K; Et N K q = F K × M R = F K × L R — F K × L M. Eſtque (ut prius in Hyperbolâ) F D × L R ſimile D K × M R, ſimile F K × L M.

Schol. Ex hoc exceſſu & Defectu quadratorum ex Ordinatis, in HYPERBOLA, & ELLIPSI, Magnus ille è veteribus Geometra APOLLONIUS *Pergæus* hæc nomina ſectionibus impoſuit, quibus uſa eſt omnis poſteritas.

Prop. XXV. Theor. XXIV.

In Parabola, ſit quælibet diameter D C, *& ad hanc* 94.
quælibet ordinata B C; *ſi interceptæ diametro* D C *& conterminæ ordinatæ* C B, *fiat tertia proportionalis* L R; *Dico quadratum ordinatæ cujuſvis ad eandem diametrum* K N *æquale eſſe rectangulo ex eadem recta* L R *& interceptâ diametro* D K, *viz.* N K q = L R × D K.

Ob D C : C B : L R ∺ erit CBq = D C × L R. Et(per *Coroll.* 4. prop. 20.)

$\left.\begin{matrix}CBq\\ DC\times LR\end{matrix}\right\}$: N K q :: D C : D K :: D C × L R : D K × L R.

Unde N K q = D K × L R ; & ſic ubique.

Def. Recta LR (ſicut in Hyperbolâ & Ellipſi) dicitur *Latus rectum* ſeu *Parameter.*

Schol. Ex eo quod quadratum ordinatæ *Comparatum* ſive *applicatum* rectangulo ex abſciſsâ & parametro, eidem æquale ſit, Hæc ſectio *Apollonio* PARABOLA dicta eſt.

Prop. XXVI. Theor. XXV.

95. 96. *In Parabola, ſi binæ quælibet diametri* C D, G E (*quarum vertices* C, G) *ſi opus productæ, rectæ cuivis* I H *utrinque ad Sectionem in* I, H *terminatæ, & ſi opus productæ, occurrant in binis punctis* D, E, *vel binis quibuſvis rectis inter ſe parallelis* I H, O P, *ad ſectionem ut prius terminatis occurrant in* D, Q, E, R ; *Si recta ſit unica, Erit,*

I D × D H : H E × E I :: C D : G E. *Sin binæ ſint rectæ,* I D × D H : O R × R P :: C D : G R ; *& ſic de cæteris punctis.*

96. *Idem erit de quadratis contingentium, ſi coeuntibus ex gr. punctis* H, I *recta unica, vel binarum rectarum alterutra fiat contingens, viz.* H D q : I E q :: C D : G E &

H D q : O R × R P :: C D : G R *& ſic de cæteris punctis.*

Biſectis I H, O P in S, T, Connexa S T occurrat ſectioni in A, erit hæc rectarum I H, O P diameter ; ductâque L A K his parallelâ occurrente diametris in L, K, eadem ſectionem in A continget ; à punctis C, G intelligantur ordinatæ ad diametrum A S T rectæ C M, G N, quæ erunt propterea rectis L A K, I H, O P parallelæ ; Erit per prop 18.

1 ID×DH : $\left\{\begin{matrix}LAq\\ CMq\end{matrix}\right\}$ :: CD : CL & per *Corol.* 4. prop. 20.

2. $NGq : CMq :: \left\{\begin{matrix}NA\\GK\end{matrix}\right\} : \left\{\begin{matrix}MA\\CL\end{matrix}\right\}$ ergo ex æquo

$ID \times DH : NGq :: CD : GK$; & per prop. 18.

3. $HE \times EI : \left\{\begin{matrix}AKq\\NGq\end{matrix}\right\} :: GE : GK$; ergo ex æquo

4. $ID \times DH : HE \times EI :: CD : GE$.

Rurſus, per prop. 18.

5. $OR \times RP : HE \times EI : GR : GE$; ergo per proport. 4. & 5. ex æquo

$ID \times DH : OR \times RP :: CD : GR$. & ſic de cæteris punctis.

Si recta D I H E ſit contingens, coeuntibus punctis H, I fit $ID \times DH = DHq$ & $EI \times EH = IEq$ &c.

Coroll. Si V X ſit parameter diametri A S T, erit GE 95. 96.
$\times VX = IE \times EH$, & ſic de punctis D, Q, R. Nam
(per hanc prop. 26.) $IS \times SH = SHq : IE \times EH ::$
$AS : GE :: VX \times AS : VX \times GE$; Sed $VX \times AS$ (per
prop. præced) $= SHq$; Ergo & $VX \times GE = IE \times$
EH *i. e.* $VX : IE :: EH : GE$. Hoc eſt, ut parameter diametri cujuſvis in Parabolâ, ad ſummam duarum
quarumlibet ordinatarum ad eandem diametrum; ſic earum differentia, ad differentiam abſciſſarum. Nam V X 95.
eſt Parameter, I E (ob $IS = SH$ & $SE = NG$) eſt ſumma ordinatarum S H, N G; E H eſt earum differentia,
Et $EG = NS$ eſt abſciſſarum A S, A N differentia.

Prop. XXVII. Theor. XXVI.

Si duæ rectæ L A, A E, *quamlibet è ſectionibus conicis,* 97. 98.
vel ſectiones oppoſitas in L, E *contingentes, vel qua-* 99. 1.
rum una A L *in hyperbola, vel ſectionibus opp. ſit* 2. 3.
forte aſymptotos, concurrant in A, *Recta vero quævis* N D *earum alteri* E A *parallela* (*ſi opus producta*) *alteri* L A (*ſi opus productæ*) *occurrat in* B, *& ſectioni, vel utrivis ſectionum oppoſitarum in* N, D, *& tactus conjungenti* L E *vel* (*ſi* L A *ſit aſymptotos*) *rectæ per tactum* E *aſymptoto* L A *parallelæ, in* R; *Dico* $BRq = BD \times BN$.

97. 98. *Idem erit de quadrato ex* BD *vel* BN, *ſi coeuntibus*
99. 1. *punctis* N, D, *recta* RBDN *ellipſin, vel unam è ſectionibus oppoſitis contingat.*

97. 98. Nam (per prop. 17. vel 18.) $BD \times BN : BLq ::$
99. 1. 2. $AEq : ALq ::$ (ob ſim. triang.) $BRq : BLq$. Ergo
3. $BD \times BN = BRq$.

3. Si (tactu L in infinitum abeunte) recta AL fiat aſymptotos, erit (ob punctum L infinite diſtans) EL parallela AL unde $BRq = AEq =$ (per *Coroll.* prop. 15.) $BD \times BN$.

97. 98. 2. Coeuntibus punctis D, N, fit $BD \times BN = BNq$
99. 1. vel $BDq =$ (prius) BRq.

Coroll. Si RBDN ſit contingens, ob $BRq = BNq$, Erit $BR = BN$ vel BD.

Prop. XXVIII. Theor. XXVII.

4. 5. *In Hyperbola vel Sectionibus oppoſitis, ſi à binis punctis* A, D *agantur duæ rectæ* AG, DF *inter ſe parallelæ, utrivis aſymptoto* (*ſi opus productæ*) *in* G, F, *occurrentes, & ab iiſdem punctis totidem aliæ* AH, DI, *inter ſe quoque parallelæ, alteri aſymptoto* (*ſi opus productæ*) *occurrentes in* H, I; *Dico* $AG \times AH = DF \times DI$.

Connexa AD, (& ſi opus producta) ſecet aſymptotos in B, E; Erit ob $AB = DE$, & $BD = EA$, & ob ſim. triangula

$$\left.\begin{matrix}AB\\ED\end{matrix}\right\} : \left\{\begin{matrix}BD\\EA\end{matrix}\right\} :: AG : DF \text{ \&}$$

$ED : EA :: DI : AH$ Ergo

$AG : DF :: DI : AH$

i, e. $AG \times AH = DF \times DI$.

6. *Coroll.* 1. Hinc ſi AG, DF ſint aſymptoto CH, & AH, DI aſymptoto CG parallelæ, erunt conſtituta parallelogramma CGAH, CFDI æqualia. Nam parallelogramma æquiangula proportionalia ſunt rectangulis è lateribus, quæ (prius) æqualia ſunt.

Coroll,

7. *Coroll.* 2. Unde Rectæ BAE, KDL hyperbolam vel ſectiones oppoſitas utcunque contingentes, ab angulo aſymptotôn abſcindunt triangula LCK, BCE æqualia. Nam æqualia parallelogramma IDFC, GAHC (ob KL, BE, in D & A biſectas) ſunt eorum dimidia

7. *Coroll.* 3. Ob æqualia triangula ECB, LKC, & angulum ad C communem, vel æqualem; Erit

1. $LC : CE :: CB : CK$. Et componendo vel dividendo.

2. $\left\{\begin{matrix} LC \pm CE \\ LE \end{matrix}\right\} CE :: \left\{\begin{matrix} CB \pm CK \\ BK \end{matrix}\right\} : CK$

7. *Coroll.* 4. Si contingentes (ſi opus productæ) concurrant in O; addito vel dempto communi ſpatio CEOK, æqualia etiam erunt triangula OKB, OLE, & angulus ad O communis vel æqualis; Ideoque

1. $BO : OE :: LO : OK$; unde comp. vel div.

2. $\left\{\begin{matrix} BO \pm OE \\ \\ BE \end{matrix}\right\} : \left\{\begin{matrix} OE \\ \text{vel} \\ BO \end{matrix}\right\} :: \left\{\begin{matrix} LO \pm OK \\ \\ LK \end{matrix}\right\} : \left\{\begin{matrix} OK \\ \text{vel} \\ LO \end{matrix}\right\}$

Et dimidiatis antecedentibus, erit

3. $BA : OE$ vel $BO :: LD : OK$ vel LO.

Et dividendo

4. $BA \left\{\begin{matrix} BO - BA \\ OA \end{matrix}\right\} :: LD : \left\{\begin{matrix} LO - LD \\ DO \end{matrix}\right\}$ &c.

Prop. XXIX. Theor. XXVIII.

8. 9. *Si recta* LT *Hyperbolam in* T *contingens utrivis aſymptoto occurrat in* L, *ab* L *vero ducatur recta* LOP, *quæ ſectioni, vel ſectionibus oppoſitis, occurrat in duobus punctis* O, P, *Et à punctis* O, T, P, *ducantur rectæ* OQ, TV, PR *aſymptoto* CL *in qua eſt punctum* L *parallelæ, alteri aſymptoto* CM (*ſi opus productæ*) *occurrentes in* Q, V, R; *Si puncta* O, P *ſint ad eandem ſectionem, Rectarum* OQ, PR *ſumma, Sin ad oppoſitas, earundem differentia, æqualis erit duplæ* TV.

Productæ ſi opus L T, L O P occurrant aſymptoto V R in A, M, & ab utrovis O, P, (ex. gr. O) ducatur O B parallela V C, occurrens L C in B.

Ob triangula M P R, O L B ſimilia, & M P = O L, erit P R = L B. Eſtque B C = O Q, unde C L = O Q + P R = (ob A T = T L) 2 T V.

Coroll. 1. Hinc ſi per idem punctum L ducantur quotlibet L O P, erit duarum quarumvis O Q, P R ab ejuſdem rectæ L O P punctis ductarum ſumma, vel differentia, duarum O Q, P R ab alterius cujuſvis rectæ L O P punctis ductarum ſummæ vel differentiæ æqualis. Nempe ubique æqualis duplæ T V.

10. *Coroll.* 2. Si Recta per L ſit alteri aſymptoton parallela, in unico puncto O ſectioni occurret; Eritque, in hoc caſu, ſola O Q = 2 T V. Nam O Q = C L = 2 T V. Unde in Corollario præcedenti eadem eſt ratio ſolius hujuſmodi O Q, quæ duarum aliarum O Q, P R ſummæ, vel differentiæ.

Prop. XXX. Probl. II.

11. 12. 13. 14. *Datæ rectæ lineæ,* D E, *ad ſectionem quamvis conicam, vel ad ſectiones oppoſitas utrinque in* D, E *terminatæ, diametrum invenire; Et centrum in Hyperbola vel opp. ſect. & Ellipſi.*

Ducatur quælibet H I ipſi D E parallela, ſectioni, vel ſectioni oppoſitæ, vel ſectionibus oppoſitis utrinque in H, I, occurrens. Biſecentur D E, H I in F, G; Erit connexa F G rectæ D E diameter, per *Coroll.* 10. prop. 20.

11. 12. Inventâ hoc modo quâlibet diametro ellipſeos, vel determinatâ hyperbolæ, hæc producta occurrat ſectioni vel ſectionibus oppoſitis in A, B. Biſecetur A B in C; Erit C centrum per prop. 22. Vel inventis hoc modo duabus quibuſvis diametris, Hæ (per eandem prop. 22.) in Centro ſe interſecant. Poſteriori methodo, datâ utriuſvis è ſectionibus oppoſitis, vel Ellipſeos aliquâ tantum portione, centrum invenitur.

Prop.

Prop. XXXI. Probl. III.

In omni ſectione conicâ, & inter ſectiones oppoſitas, A 15, 16.
dato ſectionis puncto D, *ad datam diametrum* A B, 17. 18.
ducere ordinatam D O; *Et diametrum huic conjugatam in Hyperbolâ vel ſect. opp. & Ellipſi.*

In Hyperbolâ, inter Sectiones oppoſitas, & in Ellipſi, 15. 16.
Invento centro C, connexa D C & producta occurret 17.
ſectioni, vel ſectioni oppoſitæ in E; Per E ducta E F parallela A B occurret ſectioni, vel ſectioni oppoſitæ in F, vel fortè ſectionem, vel ſectionem oppoſitam in E continget tantùm. Si occurrat in F, connexa D F occurret diametro A B in O; Eritque D O ordinata quæſita. Nam D C = C E unde ob parallelas rectas erit D O = O F. Liquet ergo propoſitum per *Coroll.* 2. prop. 22.

Si recta per E diametro A B parallela ſit contingens, *Supple figuras hujus caſus.*
erit ipſa D E ordinata quæſita. Nam (per *Coroll.* 1. prop. 22.) erit in hoc caſu recta E F parallela ordinatis ad diametrum D C E, unde (per prop. 23. & def. ſeq.) E C D, B C A ſunt diametri conjugatæ, ideoque (per ejuſdem prop. 22. *Coroll.* 2.) ad ſe mutuo ordinatæ. Inventâ vero quâlibet ordinatâ, recta per centrum huic parallela erit diameter huic conjugata.

In Parabolâ, Ductâ utcunque D A diametrum A B 18.
in A ſecante; in D A productâ, fiat A E = D A; Ducta vero E F ipſi A B parallela occurret ſectioni in F, junctaque F D occurret A B in O, Eritque D O ordinata quæſita. Nam ob rectas parallelas, & ob D A = A E, erit D O = O F; unde liquet propoſitum per *Coroll.* 13. prop. 20.

Prop. XXXII. Probl. IV.

In omni ſectione conicâ rectam D T *ducere, quæ ſe-* 19. 20.
ctionem in dato puncto D *contingat.* 21.

In

19. 20. In Hyperbola & Ellipſi, Invento centro C, ductâqu diametro D C, fiat ad hanc ordinata quælibet A O; deinde per D ductâ rectâ D T parallelâ A O, Propoſito ſatisfit. Patet per *Coroll.* 8. prop. 20.

21. In Parabola, Inventâ quâlibet diametro B C, per D agatur D E ipſi B C parallela, quæ propterea erit diameter; Ad hanc fiat quælibet ordinata A O, & per D ducatur D T ipſi A O parallela; Hæc (per idem *Coroll.* 8.) ſectionem in D continget.

Prop. XXXIII. Probl. V.

22. *Datæ Hyperbolæ vel Sectionum oppoſitarum aſymptotos invenire.*

Inventâ quâlibet diametro determinatâ C B, & centro C, ducatur utcunque F G parallela C B, unam ſectionum in G ſecans atque huic oppoſitam in F [ve ſi non detur ſectio oppoſita, ductâ diametro C M ipſ C B conjugatâ, ſecante G F in M, fiat M F = M G] Diametroque F G deſcribatur ſemicirculus F P Q G Erectâque ubicunque ad F G perpendiculari N O, æquali ſemidiametro C B, per O agatur P O Q ipſi F G parallela, ſecans circuli peripheriam in P, Q punctis Ab his, dimiſſis ad diametrum F G perpendicularibu Q R, P S, Erunt junctæ C R, C S aſymptoti.

Nam C B q = (ex conſtruct.) N O q = (ob parall.) Q R q = (propter circ.) G R × R F ; Unde (per *Coroll.* 4. prop. 16.) erit punctum R ad alteram aſymptoton Parique ratione erit punctum S ad alteram.

Prop. XXXIV. Probl. VI.

23. 24. *In omni ſectione conicâ, Datæ diametri* A B *Parametrum invenire.*

In Hyperbolâ, recta E B D in vertice B contingens & ad aſymptotos in E, D terminata, æqualis eſt (per def.)

ſecundæ

cundæ diametro ipſi A B conjugatæ. In Ellipſi diameter E D diametro A B conjugata, & ad ſectionem i E, D terminata, eſt ſecunda diameter. Inventâ ergo n Hyperbola & Ellipſi hujuſmodi rectâ E D, fiat pſis A B : E D tertia proportionalis L R; Hæc (per ef.) erit ipſius A B parameter.

In Parabolâ, Ordinatâ ad diametrum A B quâvis ectâ G I, Ipſis A I, I G, fiat tertia proportionalis L R; rit L R (per def.) diametri A B parameter. 25.

Def. Duæ ſectiones Conicæ ſe mutuò *tangere* di- 26.
untur, Si in communi puncto B eadem recta A B tramque ſectionem contingat.

Prop. XXXV. Theor. XXIX.

In ſectionis cujuſvis conicæ vel ſectionum oppoſitarum, quatuor quibuſvis punctis A, B, C, D, *concurrant quatuor rectæ infinitæ* A B, A C, B D, C D, *ſcilicet in unoquoque duæ; Per quintum verò quodvis ſectionis vel utriuſvis ſect. opp. punctum* E *tranſeant duæ rectæ infinitæ* E N, E H, *priorum duobus quibuſvis* A B, A C *in aliquo quatuor punctorum* A *concurrentibus reſpectivè parallelæ, quarum* E N *occurrat* A C *in* Q, D B *in* N; E H *vero occurrat* A B *in* R, C D *in* H. 27. 28. 29.

His poſitis; Si, manentibus punctis A, B, C, D, *punctum* E, *cum rectis* E N, E H *priorem parallelismum ſervantibus, per totam ſectionem vel ſect. opp. ſucceſſivè transferri intelligatur, Erunt in omni ejuſdem puncti* E *ſitu rectangula* E R × E H, E N × E Q *ad invicem in eadem ratione.*

Manentibus vero punctis A, B, C, *&* E, *punctoque* D (*hoc eſt rectarum* B D, C D *interſectione*) *per totam ſectionem vel ſect. opp. tranſlato, Erunt in omni ejus ſitu partes* E H, E N *ad invicem in eadem ratione.*

Per D & utrumvis C, B, *ex. gr.* B, agantur rectæ F D G, B *n d* ipſi A C parallelæ, quarum B *d* occurrat denuò ſectioni, vel ſect. opp. in *d*, rectæ vero E N in *n*, & F D G occurrat ſectioni, vel ſect. opp. in F, & rectæ A B in G; connexaque C *d* occurrat D G in S, E H in *h*.

Porrò recta H E R occurrat denuo ſectioni, vel ſect. opp. in T; ductâque rectarum A C, B *d* diametro M K, (biſecante ſcilicet A C, B *d*, in M, K,) biſecabit hæc tam D F, T E, ad ſectionem, vel ſect. opp. quàm S G, *h* R ad rectas A B, C *d* terminatas; unde S D = F G, & *h* T = E R; Idemque conſequetur, ſi forte rectarum D G, E H una vel utraque ſit contingens.

Ob ſim. triang. & rectas parallelas, erit

$$Hh : SD :: hC : SC :: AR : AG, \text{ unde}$$

$$Hh : \left\{\begin{matrix} AR \\ EQ \end{matrix}\right\} :: SD : AG. \text{ Rurſus}$$

$$\left.\begin{matrix} Bn \\ ER \end{matrix}\right\} : Nn :: DG : GB. \text{ Ergò ductis \&c.}$$

$$ER \times Hh : Nn \times EQ :: \left\{\begin{matrix} DG \times SD \\ DG \times FG \end{matrix}\right\} : GB \times AG :: (\text{per prop. 17. \& 18.}) \left\{\begin{matrix} ER \times RT \\ ER \times Eh \end{matrix}\right\} : \left\{\begin{matrix} RA \times RB \\ EQ \times En \end{matrix}\right\}$$

Et comp. vel div. pro vario ſitu punctorum H, *h*; N, *n*, erit

$$\left.\begin{matrix} ER \times Hh \pm ER \times Eh \\ ER \times Eh \pm ER \times Hh \\ ER \times EH \end{matrix}\right\} : \left\{\begin{matrix} Nn \times EQ \pm En \times EQ \\ En \times EQ \pm Nn \times EQ \\ EQ \times EN \end{matrix}\right\} :: ER \times Eh : EQ \times En :: (\text{priùs})\ DG \times FG : GB \times AG.$$

At manentibus punctis A, B, C, D, ratio D G × F G, : G B × A G (puncto E utcunque ſitum mutante) manet eadem, ergo & ratio E R × E H : E Q × E N.

2. Ob E R × E H : E Q × E N : : E R × E *h* : E Q × E *n*, erit E H : E N : : E *h* : E *n*; Manentibus vero punctis A, B, C, & E, (puncti D ſitu utcunque mutato) manet ſemper ratio E *h* : E *n*, proindéque & E H : E N eadem.

30. Si punctum A puncto B vel C coincidat, hoc eſt, ſi utravis A C vel A B, *ex. gr.* A B, ſit contingens, incidet punctum *d* in C, & *d* C, hoc eſt S *d h* evadet contingens, rectæque *d* B vel A C diameter per contingentium S *d h*, G A R occurſum tranſibit, vel erit iis parallela,

rallela, & biſecabit SG, bR; DF, TE, ut priùs; Eritque demonſtratio omnino eadem.

Si punctum D puncto C vel B coincidat, hoc eſt, *Supple fig. hujus caſus.* ſi DH *ex. gr.* ſit contingens, Quod in alio quovis ſitu puncti D, ex utrâque puncti C vel B parte, obtinere oſtendimus, in hoc intermedio obtinebit.

Si A, C & B, D, vel A, B & C, D ſimul coincidant, 31. *Supple reliquos caſus.* hoc eſt, ſi utraque AB, CD *ex. gr.* ſit contingens, in tactus conjungentem coaleſcent AC, BD, B*d*; & coincidentibus punctis Q, N, *n*, & H, *b*, fit $EQ = EN = En$, & $EH = Eb$; Hoc eſt, manente utroque contactu, erit ratio $ER \times EH : ENq$ in quovis ſitu puncti E eadem, Et ratio EH, EN, (manente E) eadem quæ in alio quovis ſitu puncti D viz. $Eb : En$.

Si punctum D infinitè diſtet, hoc eſt, ſi BD, CD 32. ſint utrivis aſymptoto hyperbolæ [vel ſect. opp.] parallelæ, vel Parabolæ diametri, utraque pars propoſitionis etiamnum valebit; Et ſi punctum A infinitè diſtet, hoc eſt, ſi BA, CA, adeoque EN, EH ſint hyperbolæ 33. aſymptoto parallelæ, vel parabolæ diametri, abibunt puncta Q, R in infinitum, & coaleſcent rectæ EN, EH; Erit vero ratio $EH : EN$ in quovis ſitu puncti D eadem. Nam quod in quibuſvis ex utrâque parte, adeoque in infinite vicinis rectarum BD, CD vel BA, CA, poſitionibus obtinere oſtendimus, in hiſce intermediis (nec diverſis) obtinebit.

Coroll. 1. Poſitis ut priùs, Si, connexâ BC ſecante 34. *pro omnibus caſibus.* EH in *b*, fiat $EH : Eb :: EN : En$, (puncto *n* ſumpto ad eaſdem partes puncti E reſpectu puncti N, ad quas jacet *b* reſpectu H,) & connectatur B*n*, eadem ſectionem in B continget. Nam accedente puncto D ad B, recta DB evadet contingens, accedetque ſimul punctum H ad *b*, adeoque (ob rationem $EH : EN$ conſtantem) N ad *n*; hoc eſt, erit B*n* contingens.

Coroll. 2. Poſitis quæ priùs, ſi in rectis EN, EH ſu- 35. *pro omnibus caſibus.* mantur puncta *n*, *b* ad eaſdem puncti E partes ad quas jacent N, H reſpectivè, ita ut ſit $EN : EH :: En : Eb$, Connexæ B*n*, C*b* ſeſe mutuò in ipſa ſectione interſecabunt, aut ſaltem erunt alterutri aſymptotôn hyperbolæ, aut parabolæ diametris parallelæ. Sit earum occurſus (ſi modò aliquis) Δ, ſitque alterutrius *ex. gr.* C *b* cum ſectione occurſus (ſi modò occurrat) δ, & connectatur δ B; Hæc (propter rationem E H : E N conſtantem) non alibi quàm in *n* ſecabit E N, ideoque non erunt puncta Δ, δ ab invicem diverſa. Sin C *b* ſectioni vel ſect. opp. non ampliùs occurrat, hoc eſt, ſi ſit hyperbolæ aſymptoto vel parabolæ diametris parallela, huic parallela ex B (per hanc prop.) non alibi quàm in *n* ſecabit E N, hoc eſt erit B *n* pariter aſymptoto hyperb. vel parab. diametris parallela.

36. *Supple reliquos caſus.* *Coroll.* 3. Sint puncta quinque B, C, D, Δ, E ad ſectionem conicam vel ad ſectiones oppoſitas; Si per E *ex. gr.* tranſeat recta E N *n*, occurrens connexis B D, B Δ in N, *n*; & alia E H *b* occurrens connexis C D, C Δ in H, *b* reſpectivè, ita ut ſit E H : E N : : E *b* : E *n*; agantur verò C A, B A ipſis E H, E N reſpectivè parallelæ; Hæ aut ſibi mutuò occurrent in ipſa ſectione [vel unâ ſect. opp.] vel ſaltem (in unam coaleſcentibus rectis E N, E H) erunt uni aſymptotôn hyperbolæ vel parabolæ diametris parallelæ. Sit earum occurſus, (ſi modò aliquis) A, occurrat vero B A ſectioni, vel ſect. opp. denuò (ſi modò occurrat) in *a*, connexâque C *a*, ducatur E L parallela C *a*, occurrens C D, C Δ in L, *l* reſpectivè; Eritque (ex hâc prop.) E N : E *n* : : E L : E *l* : : (ex hyp.) E H : E *b*, quod fieri non poteſt (propter C D, C Δ non parallelas) niſi rectæ E H, E L, hoc eſt, puncta H, L; *b*, *l*, coincidant; adeoque ipſis E H, E L parallelæ rectæ C A, C *a*, hoc eſt, puncta A, *a*.

37. *Supple reliquos caſus.* Si B A ſit aſymptoto, vel parabolæ diametris parallela, ut non ampliùs occurrat ſectioni, ductis C *a*, E L eidem parallelis, oſtendetur eodem modo E H, E L, ſibi invicem, & rectæ E N, coincidere, adeoque & C A, C *a*, hoc eſt B A, C *a* eſſe aſymptoto, vel parab. diam. parallelas.

Supple hos caſus. Si ex D, Δ unum vel utrumque infinitè diſtet, vel coincidentibus C, D, vel C, △, B, D vel B, △, ex rectis connectentibus una vel duæ ſint contingentes, Eadem prorſus eſt demonſtratio.

Coroll. 4. Poſitis quæ in *Coroll.* 3. Cùm punctum A (modo infinitè non diſtet) ſit ſemper ad ſectionem, Incidente A in B, manifeſtum eſt fore A B contingentem. Idem intellige de A C, incidente A in C. *Supple hunc caſum.*

35. *Coroll.* 5. Sectio conica [vel ſect. opp.] ſectioni conicæ [vel ſect. opp.] non occurrit ad plura puncta quàm quatuor, niſi in totum congruat; Quoties autem unius hyperbolæ [vel ſect. opp.] aſymptotos fuerit alterius aſymptoto, vel parabolæ diametris parallela, aut duarum parabolarum communes fuerint diametri, id pro communi (infinitè ſcilicet diſtante) puncto cenſebitur, ſectionum vero quilibet contactus pro duobus punctis. Nam ſi dixeris eſſe quinque communia puncta A, B, C, D, E; Connexis A B, A C; B D, C D, ductiſque E N, E H (ut priùs,) ſumptoque in utràvis ſectione puncto quovis △, & connexâ B △ ſecante E N in *n*, & C △ ſecante E H in *h*, erit (ex hâc prop.) E N : E H : : E *n* : E *h*; unde (per *coroll.* 2.) punctum △ erit pariter ad alteram ſectionem. Et ſic ubique. *pro omnibus caſibus.*

Eadem eſt demonſtratio in cæteris caſibus, ſi pro infinitè diſtante puncto ducantur B A, C A, vel B D, C D aſymptotis, vel parabolarum diametris, vel utriſque (prout caſus fuerit) parallelæ; pro contactu vero concipias duorum occurſuum puncta A, & B, ſive A & C, vel B & D, ſive C & D, in unum coire in B vel C, & à communi rectâ contingente connecti, cæteraque (juxta præcedentia) fieri perinde ac ſi quinque eſſent diverſa puncta. *Supple hos caſus.*

Quamvis hæc demonſtratio manifeſte requirat, ut punctum E (ratione muneris ſui) nec alii cuivis puncto coincidat, nec infinitè diſtet, ut non directè extendatur ad caſum ubi fortè dixeris Duos eſſe contactus, & unum præterea occurſum ſed infinite diſtantem; Nihilominùs in hoc etiam caſu valet corollarium. Nam quod de duobus contactibus & occurſu quantumvis longinquo oſtenſum eſt, in hujuſmodi occurſu infinite diſtante pariter obtinebit.

Coroll. 6. De duarum ſectionum contactibus in corollario præcedenti oſtenſa de duabus hyperbolis [ve ſect. oppoſitis] unam vel utramque aſymptoton communem habentibus, pariter intelligenda ſunt. Nam quemadmodum recta conjungens duo puncta (eorum uno in infinitum abeunte) evadit parabolæ diameter vel hyperbolæ aſymptoto parallela; proindeque hujuſmodi rectarum paralleliſmus ſimplici occurſui æquipollet, ut ex ſupradictis manifeſtum eſt; ita rect utramque ſectionem contingens, hujuſmodi tactu in infinitum abeunte, fit communis aſymptotos; Et qu de contactu quantumvis longinquo vera ſunt, de communi aſymptoto pariter vera erunt.

38. *Coroll.* 7. Etiam ſi duæ Parabolæ habeant communes diametros, & duo præterea communia puncta A, B ad aliud punctum ſibi mutuò non occurrent, niſi in totum congruant. Idem intellige de uno puncto qu ſit contactus. Nam ſi dicas aliud eſſe commune punctum C, connexâ AB, ductâque huic parallelâ CE*e* ſecante unam ſectionem in E, alteram in *e*, Si biſect AB in H, ducatur HI communibus diametris parallel Erit hæc (per *coroll.* 16. prop. 20.) utriuſque ſection diameter, & biſecabit tam CE, quam C*e* in I; und coincident E, *e*; Eruntque adeò præter communes diametros quatuor communia puncta. Simili modo (concipias A, B concidere, & à communi rectâ contingente AB connecti, & diametrum HI per contactum duci) oſtendetur in ſecundo caſu, præter communes diametros, tria eſſe communia puncta quorum unum eſt contactus.

Si in priore caſu diameter rectæ AB tranſit per C erit recta per C parallela AB communis contingens eruntque in hoc caſu, præter communes diametros tria communia puncta quorum unum eſt contactus. Patet ergo ubique per *coroll.* 5.

Corol

Coroll. 8. Parabola parabolam, cui in totum non congruit, in duobus punctis non tangit. Nam recta connectens communium rectarum contingentium occurſum, & medium tactus conjungentis punctum eſſet communis utriuſque ſectionis diameter: Unde patet per *coroll.* 7.

Prop. XXXVI. Theor. XXX.

39. *pro omnibus caſibus.*

Si per quatuor puncta ſectionis conicæ [vel ſect. opp.] A, B, C, D *ducantur quatuor rectæ* A B, A C, B D, D C, *eodem modo quo in propoſitione præcedenti, Et à quovis alio ſectionis puncto* E *agantur quatuor rectæ* E L, E K, E O, E M *ad priores quatuor in* L, K, O, M, *reſpectivè terminatæ, & ad eaſdem quomodocunque inclinatæ; Item ab alio quovis ſectionis puncto* e, *agantur quatuor aliæ* e l, e k, e o, e m, *ad eaſdem rectas cum prioribus reſpectivè terminatæ, prioribuſque reſpectivè parallelæ; Erunt rectangula* E L × E M, e l × e m, *ſub rectis ſcilicet ad duas* A B, C D *in uno aliquo quatuor punctorum non conccurrentes terminatis, ad invicem, ut rectangula* E K × E O, e k × e o *ſub rectis ad reliquas duas* A G, B D *terminatis.*

Per E, *e* actis E Q N, *e q n*; E R H, *e r h*, duabus A B, A C, in eodem puncto A concurrentibus, reſpectivè parallelis; quarum E Q N, *e q n*, occurrant A C in Q, q; B D in N, *n*; E R H, *e r h* occurrant A B in R, *r*; C D in H, *h*; Erit ob ſim. triang.

ER : *er* :: EL : *el*, &
EH : *eh* :: EM : *em*, ductiſque &c.
1. ER × EH : *er* × *eh* :: EL × EM : *el* × *em*.
Rurſus, EQ : *eq* :: EK : *ek*, &
EN : *en* :: EO : *eo*, ductiſque &c.
2. EQ × E*n* : *eq* × *en* :: EK × EO : *ek* × *eo*.
Sed (per prop. præced.) eſt
ER × EH : *er* × *eh* :: EQ × EN : *eq* × *en*;
Ergo per proport. 1 & 2,
EL × EM : *el* × *em* :: EK × EO : *ek* × *eo*.

Si

Supple hos caſus. Si rectarum quatuor puncta conjugentium una (coeuntibus duobus punctis) ſit contingens, vel ſi duæ aliquæ ſint contingentes, & duæ reliquæ in tactus conjungentem coaleſcant; vel è quatuor punctis unum infinitè diſtet; in iis nempe caſibus in quibus obtinet propoſitionis præcedentis pars prior, unde hæc pendet, manet eadem demonſtratio. Quæ etiam ſi rectarum E L, E K &c, *e l*, *e k* &c. aliquæ vel omnes in directum jacent, (punctorum coincidentium ratione habitâ) non proinde immutabitur.

Supple hos caſus. Etiam, ſi in hyperbolâ [vel ſect. opp.] ex quatuor punctis, uno ſecundum unius aſymptoti directionem infinitè diſtante, aliud adhuc (quod priori non connexum intellige) ſecundum alterius aſymptoti directionem infinitè diſtet; vel ſi connectens duo aliqua (coeuntibus punctis, & ſimul in infinitum abeuntibus) fiat aſymptotos, duobus reliquis (ſive coeant, ſive non) finitè diſtantibus; Nihilominus in his, perinde ac in prioribus, caſibus valebit propoſitio. Nam quod in horum punctorum ſitu quantumvis longinquo obtinet, in infinitè diſtante ſitu pariter obtinebit.

40. *pro omnibus caſibus.* *Coroll.* Si quatuor rectarum duæ aliquæ in uno quatuor punctorum non concurrentes, *ex. gr.* C D, A B concurrant in X, & reliquæ duæ C A, B D in Y, & rectæ omnes E L, E K, &c. *e l*, *e k* &c. in directum jacentes fiant una recta E *e*, quæ etiam per X, Y tranſeat; erit E X : *e* X :: E Y : *e* Y. Nam in hoc caſu coincidun X, L, *l*, M, *m*; uti etiam Y, K, *k*, O, *o*; Unde

$$\left.\begin{matrix}EL\times EM\\ EXq\end{matrix}\right\} : \left\{\begin{matrix}el\times em\\ eXq\end{matrix}\right\} :: \left\{\begin{matrix}EK\times EO\\ EYq\end{matrix}\right\} : \left\{\begin{matrix}ek\times e\\ eY\end{matrix}\right.$$

ideoque : E X : *e* X : : E Y : *e* Y.

41. *pro omnibus caſibus.* Idem erit, ſi, coeuntibus punctis *ex. gr.* A, B & C, D rectæ A B, C D fiant contingentes, & reliquæ A C, B D in tactus conjungentem coaleſcant, rectaque E *e* pe contingentium occurſum X tranſiens ſecet tactus conjungentem in Y. Siquidem in hoc quoque caſu coincidunt puncta X, L, *l*, M, *m*, & Y, K, *k*, O, *o*.

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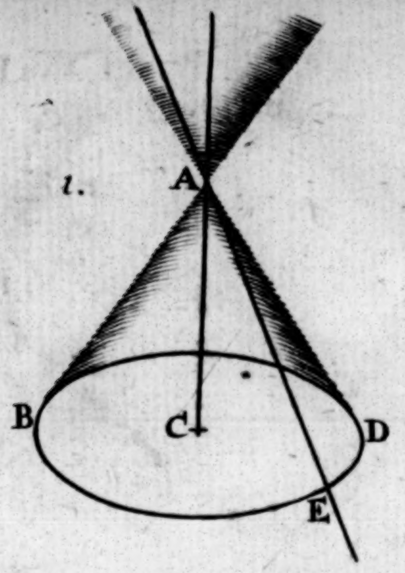

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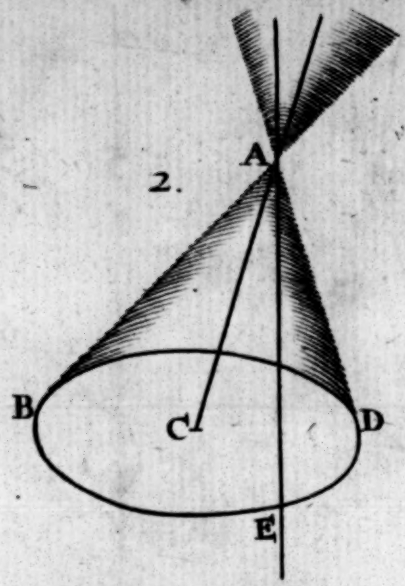

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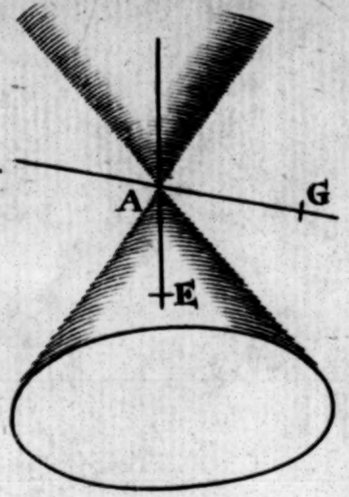

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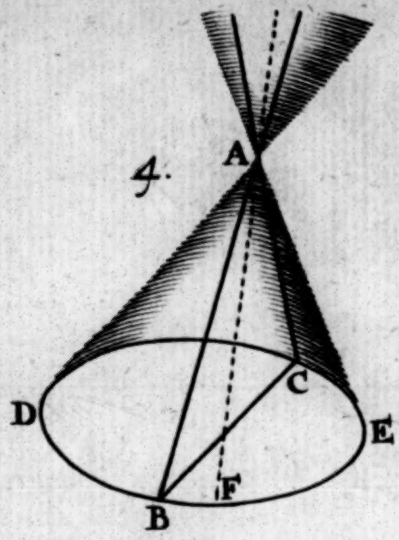

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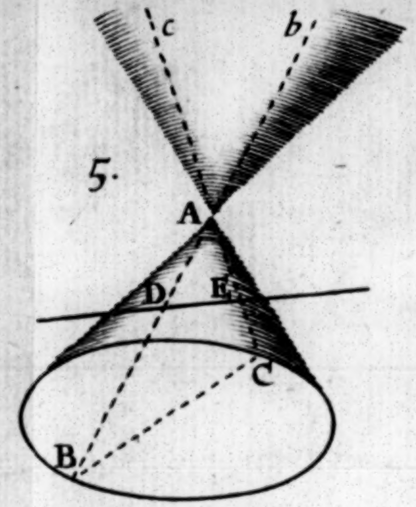

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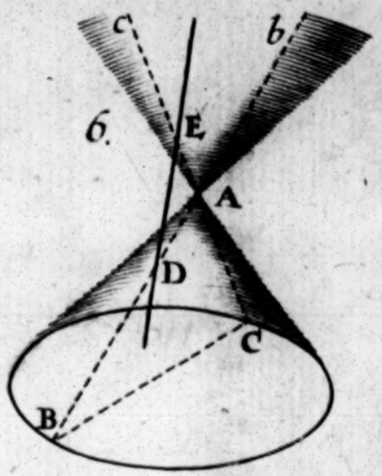

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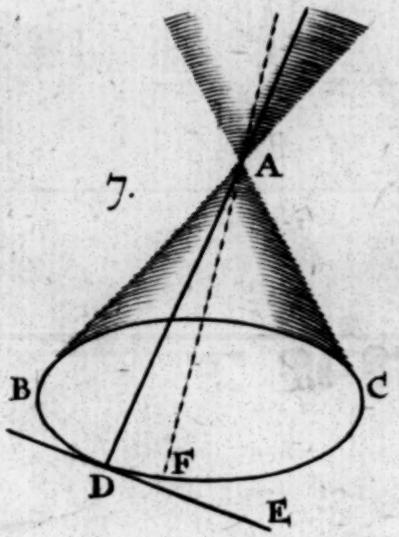

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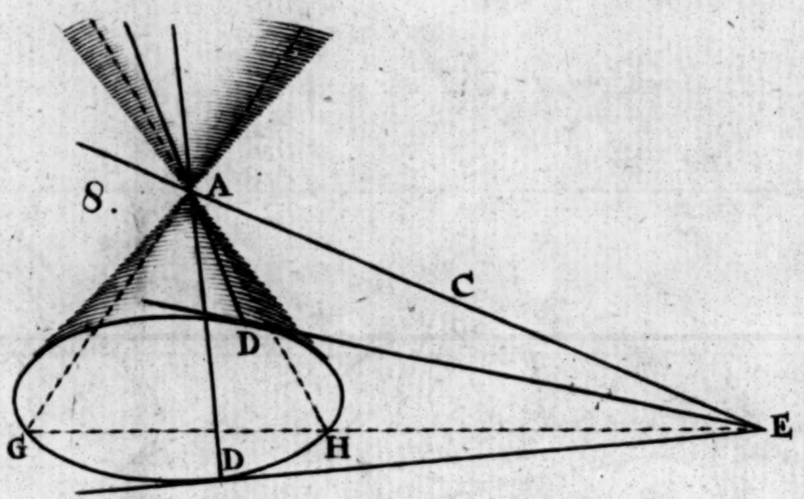

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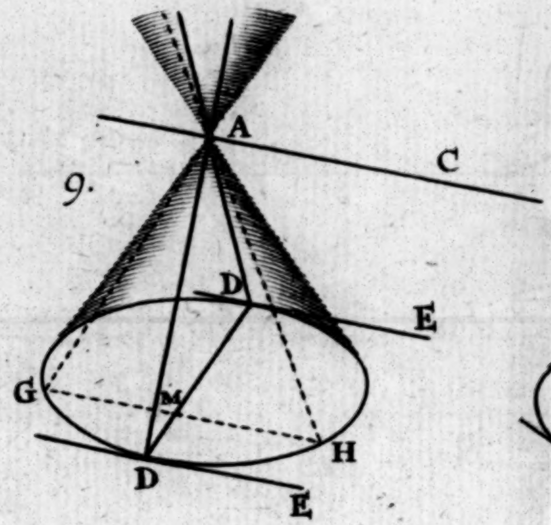

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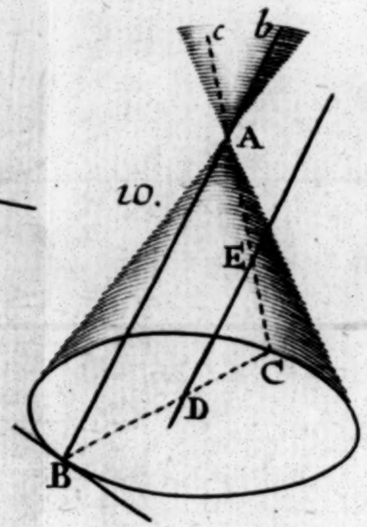

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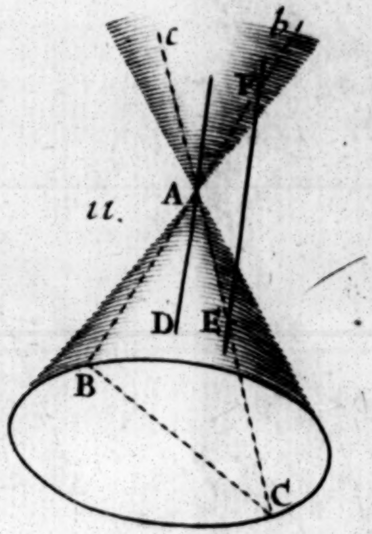

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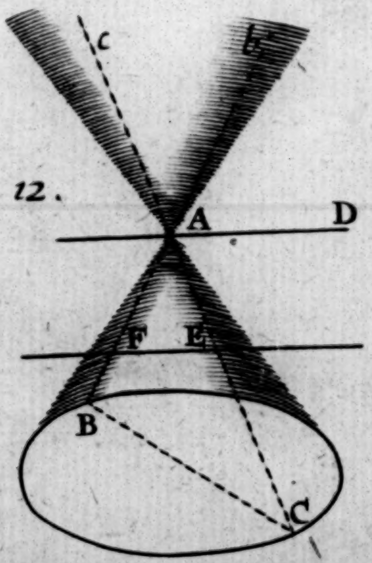

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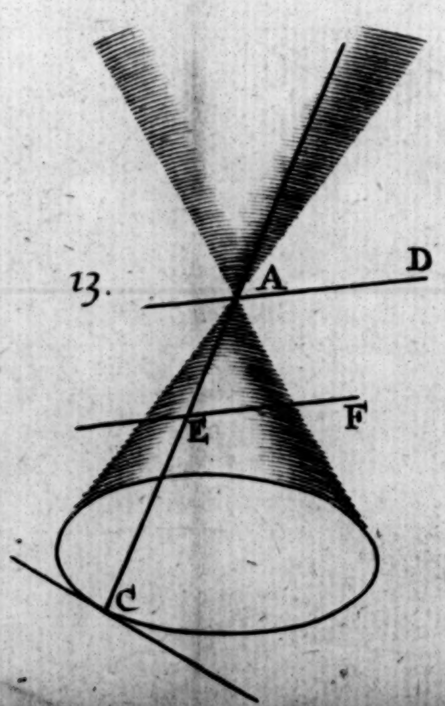

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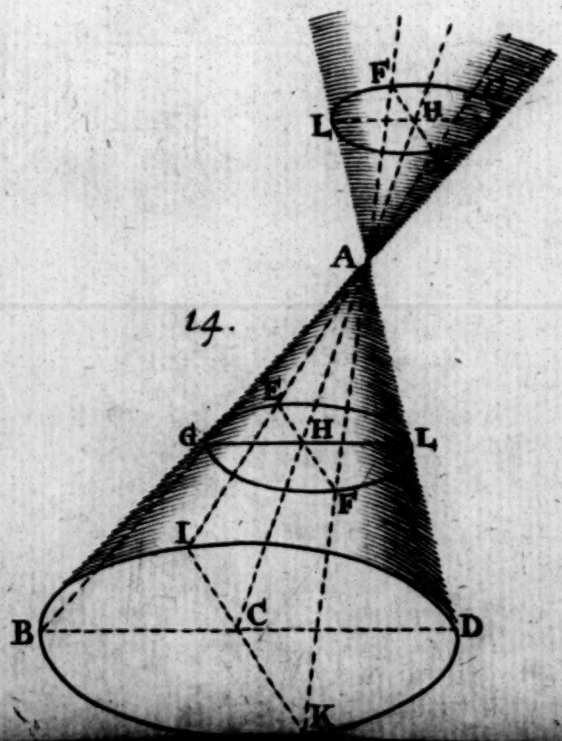

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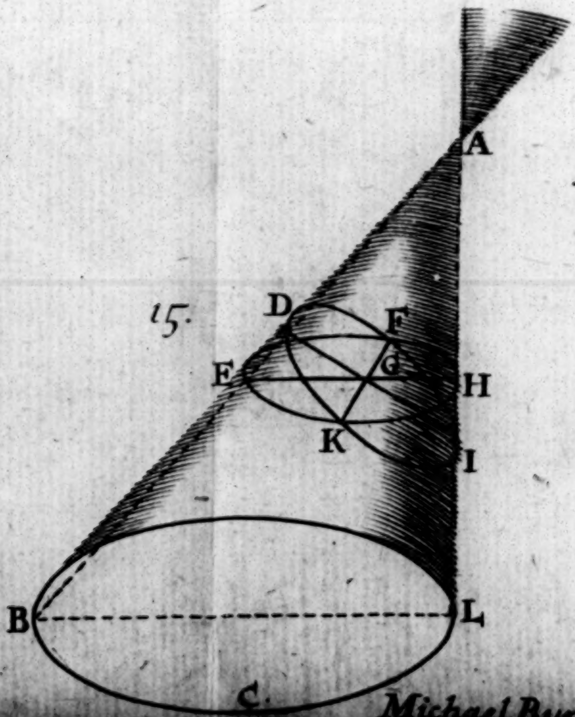

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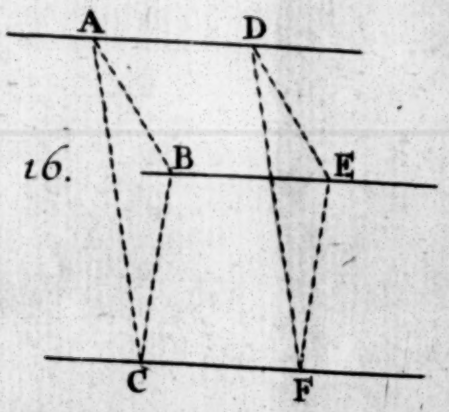

Michael Burghers sculpt.

PARS. I. TAB. II.

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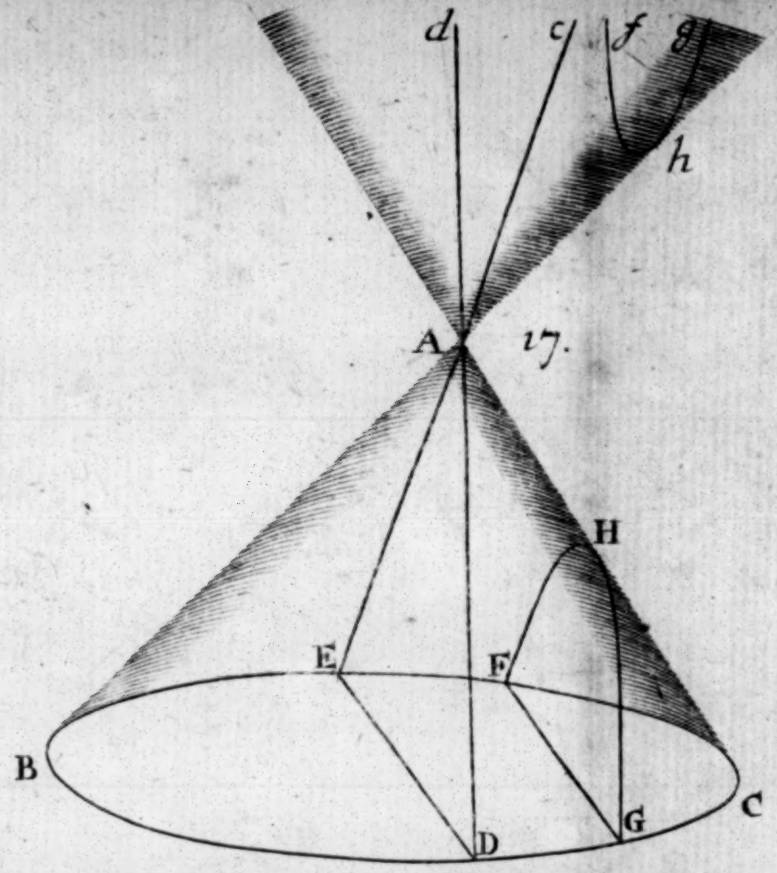

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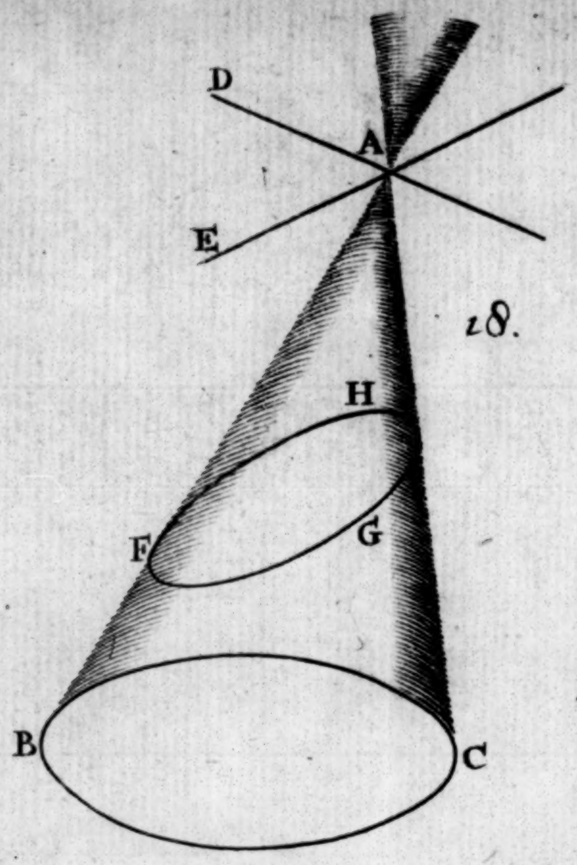

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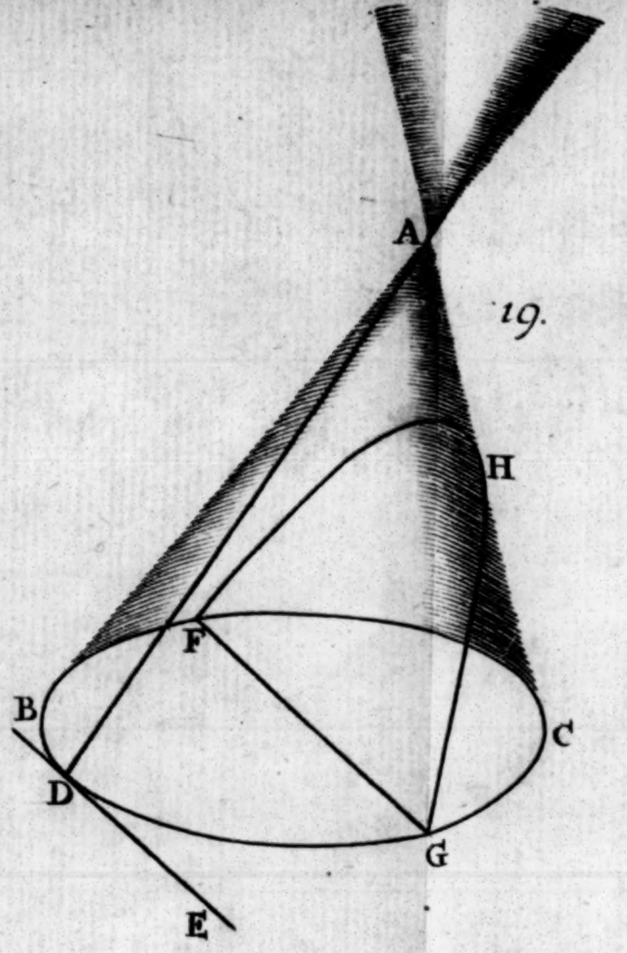

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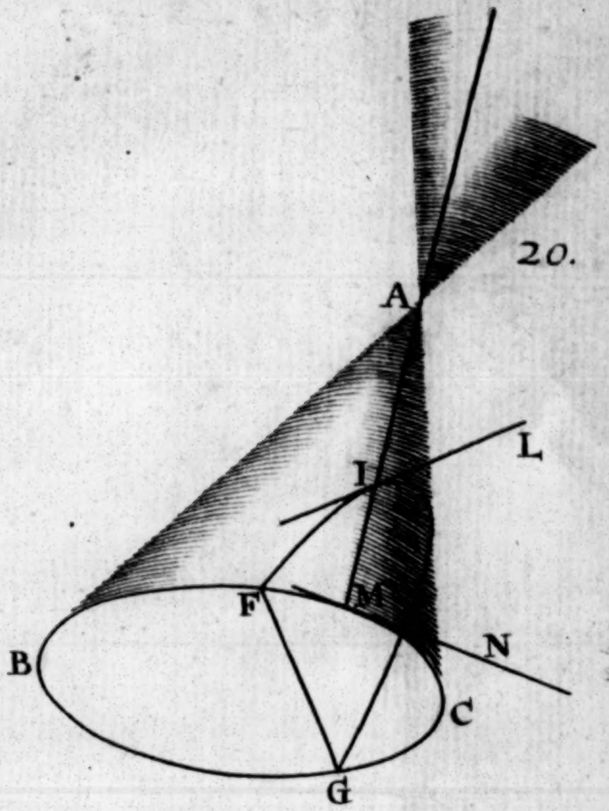

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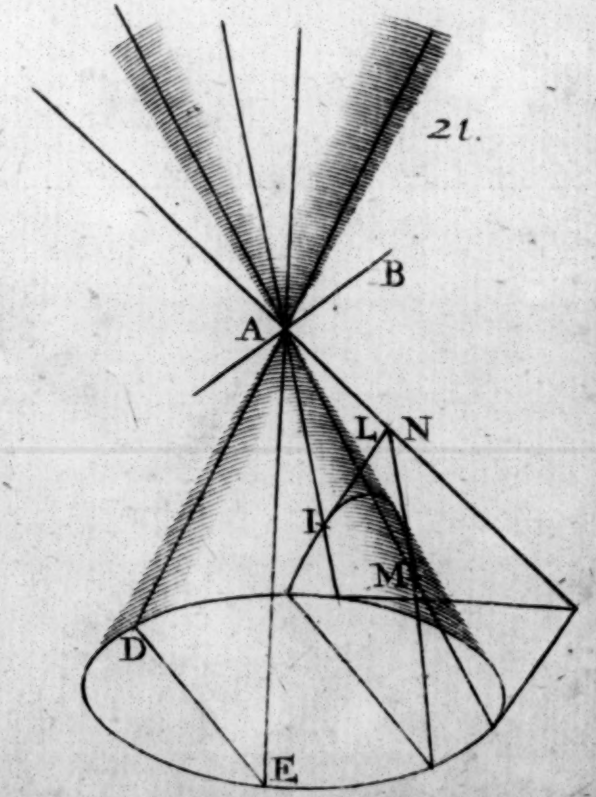

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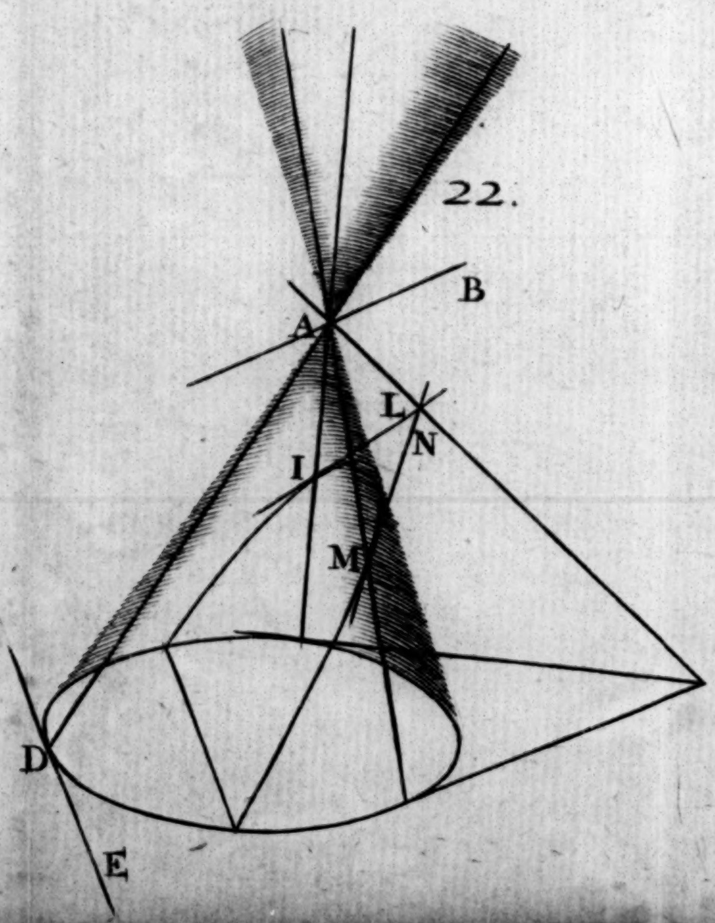

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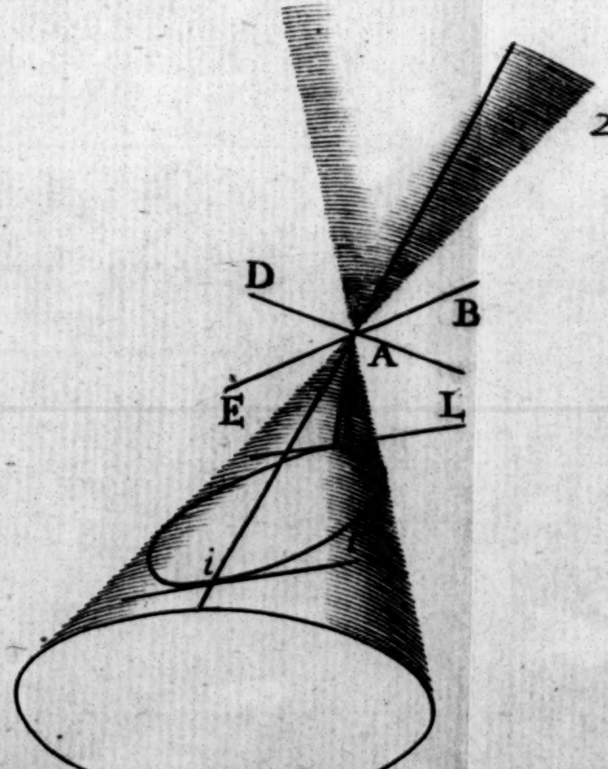

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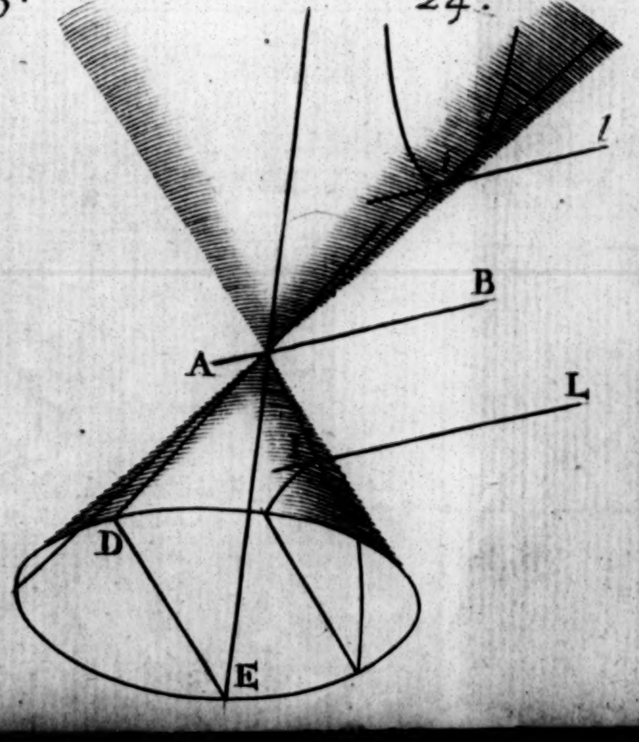

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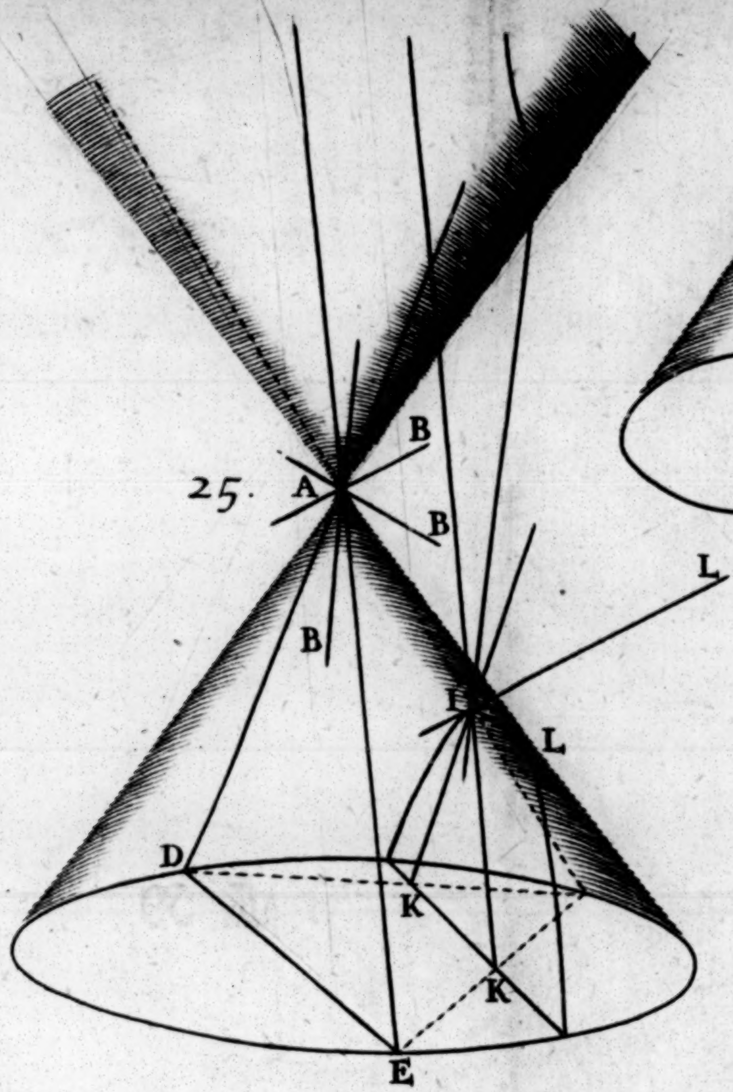

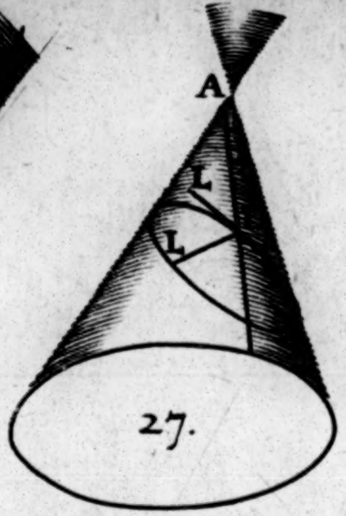

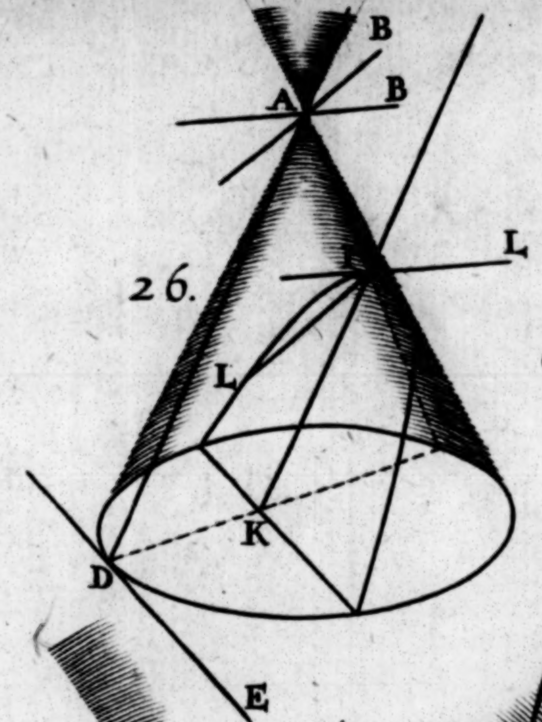

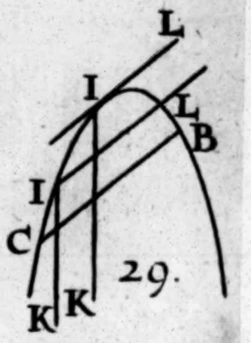

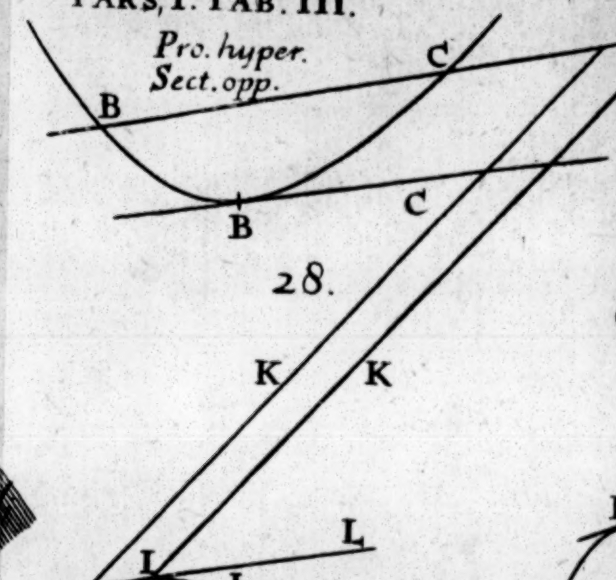

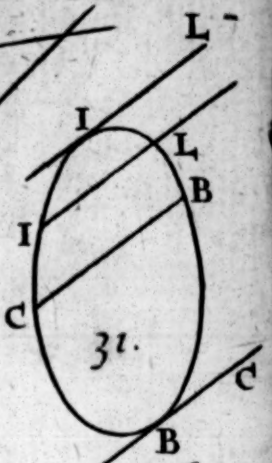

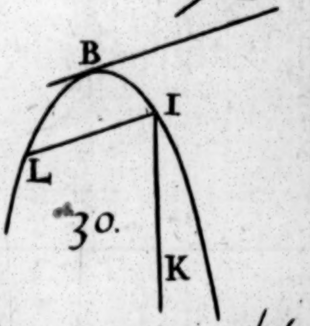

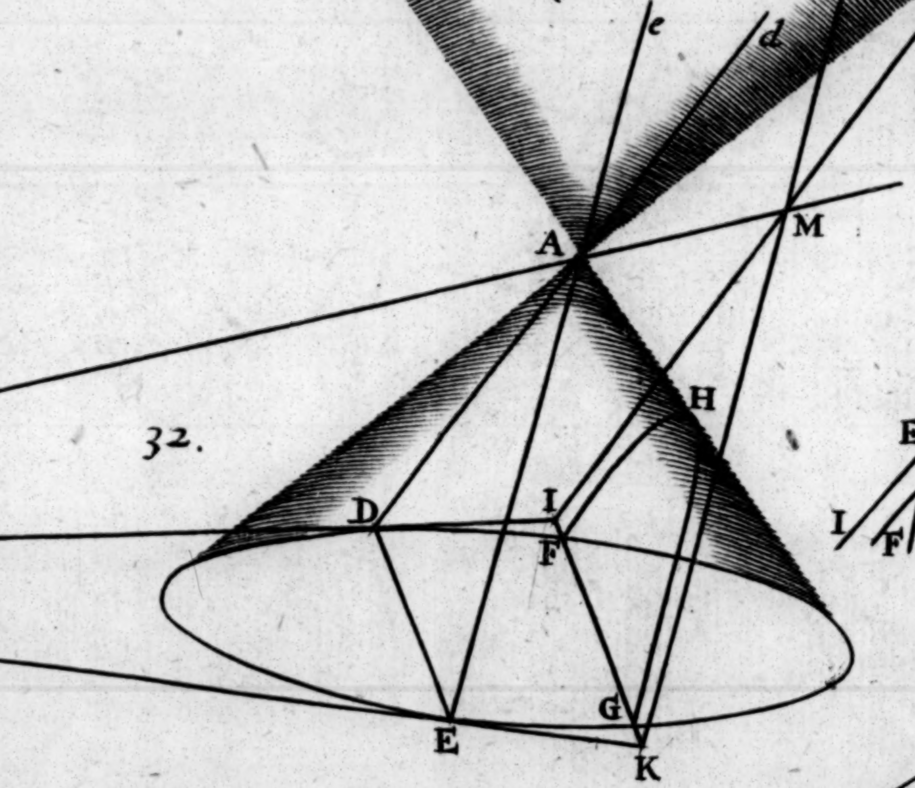

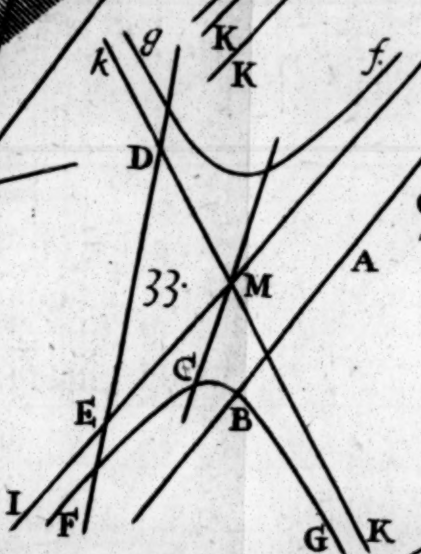

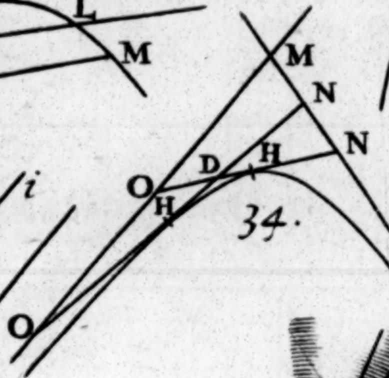

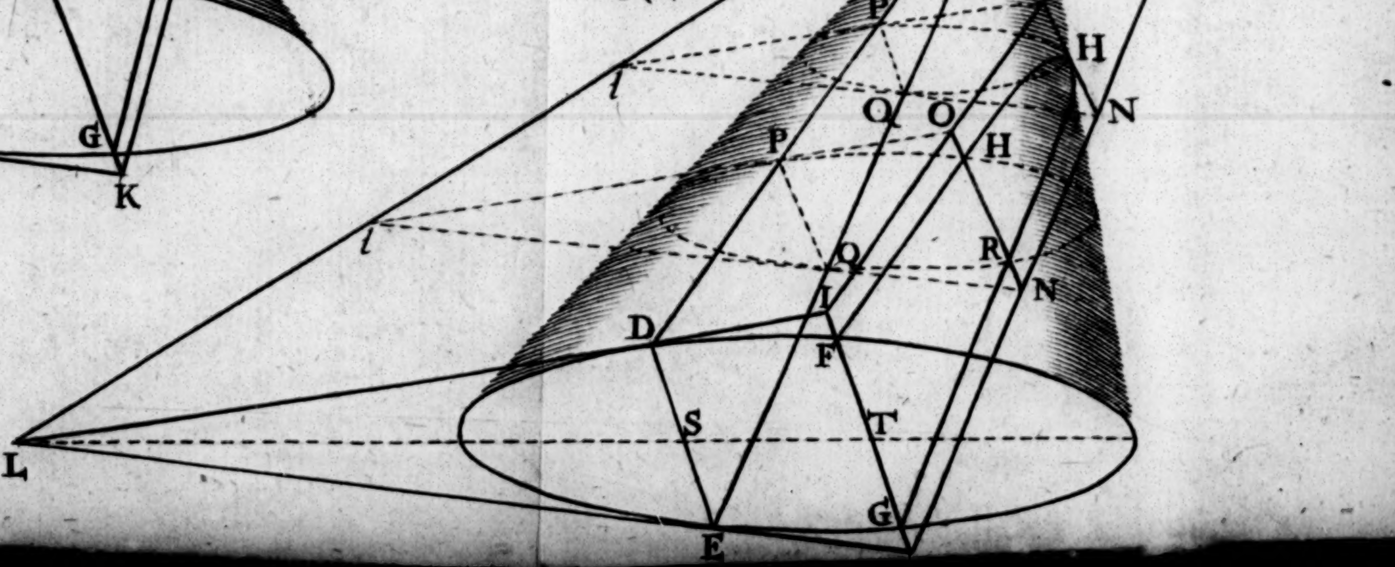

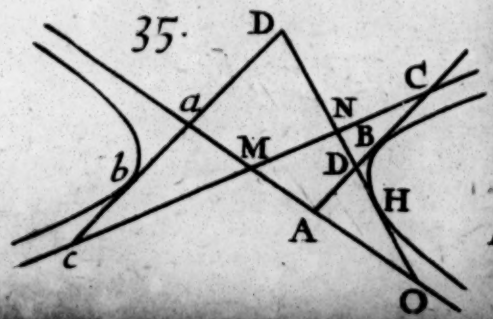

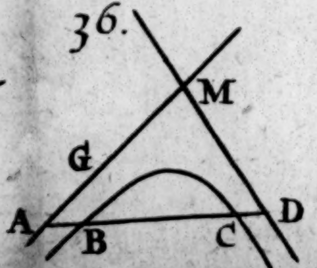

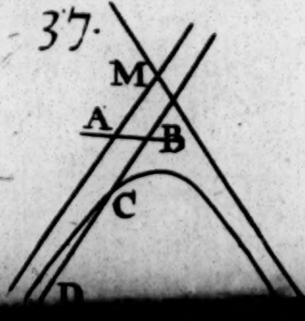

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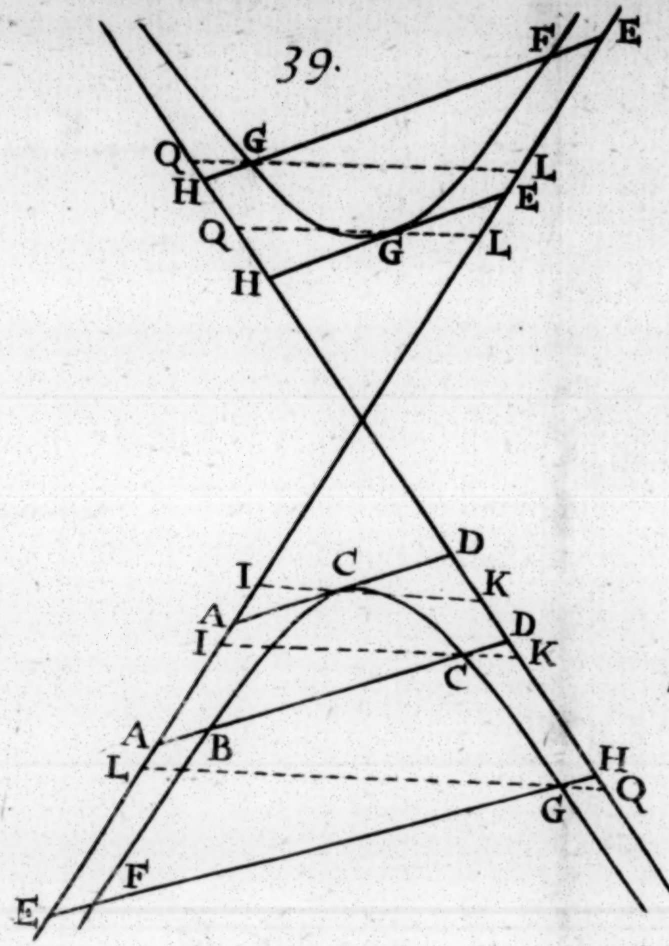

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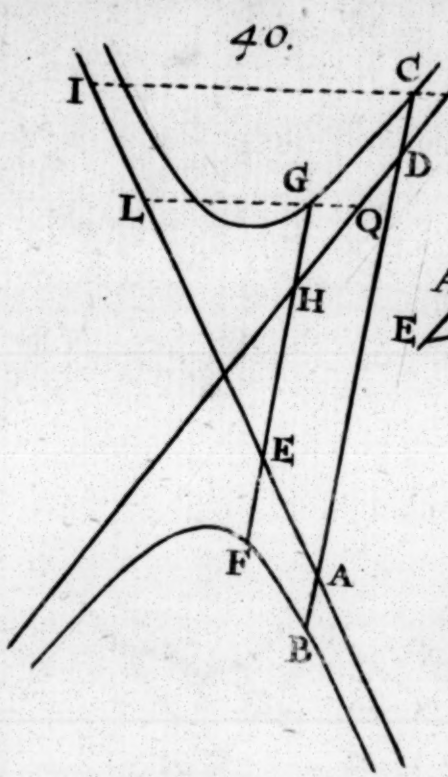

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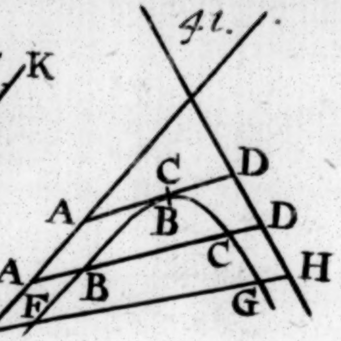

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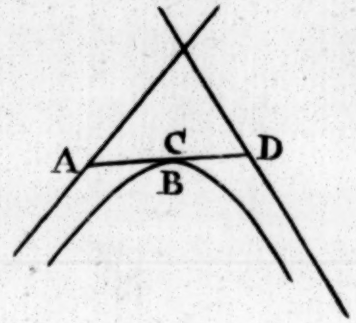

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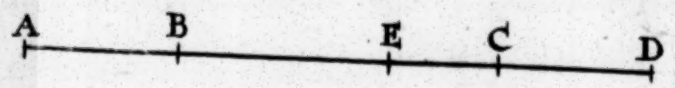

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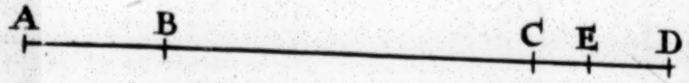

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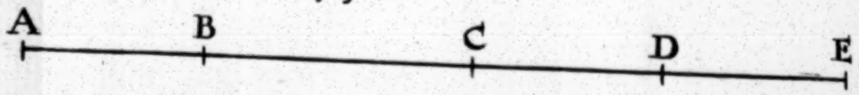

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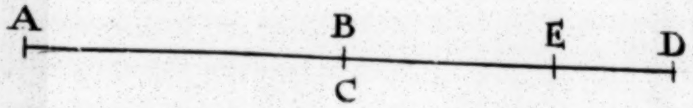

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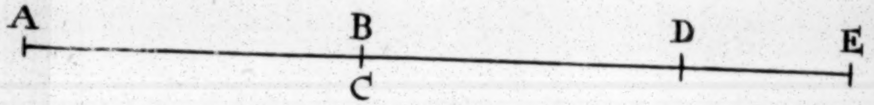

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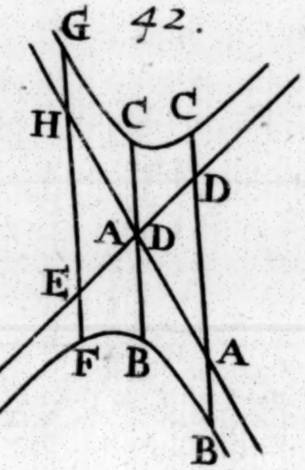

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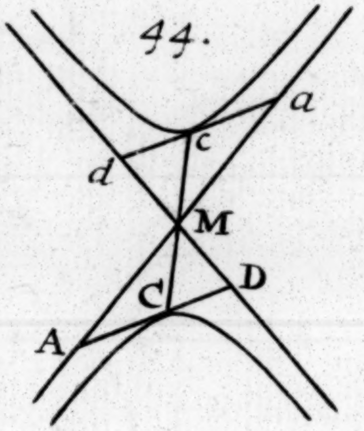

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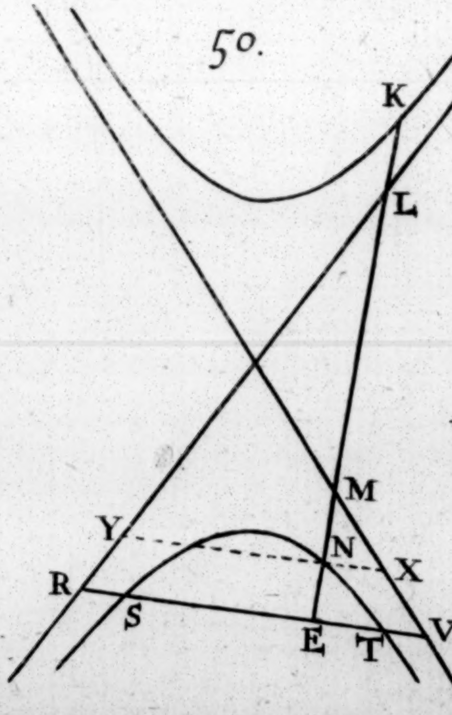

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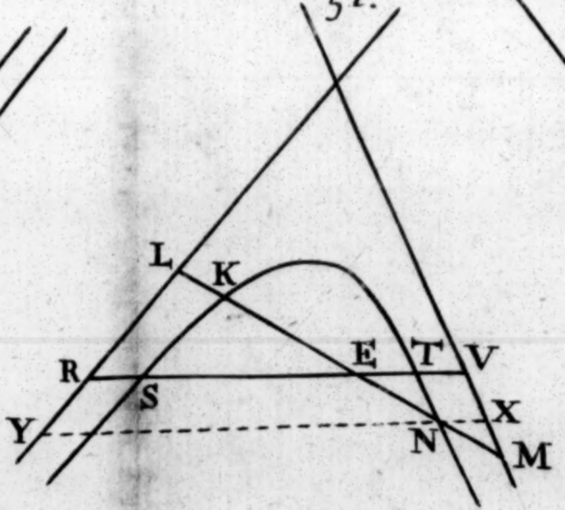

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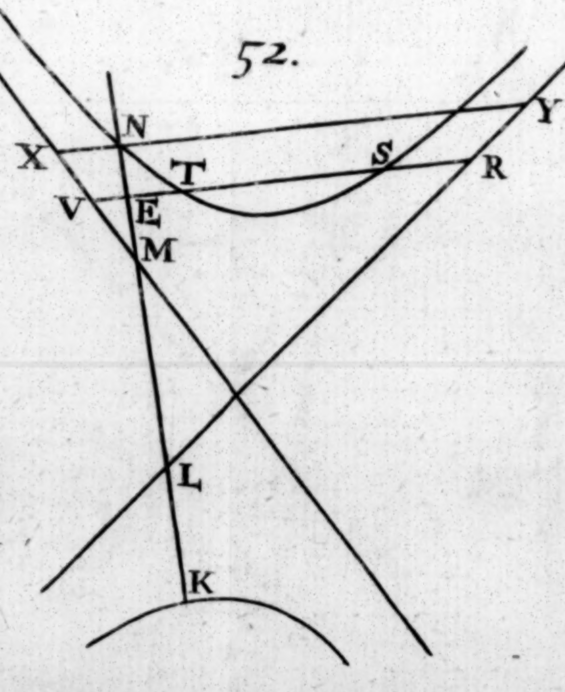

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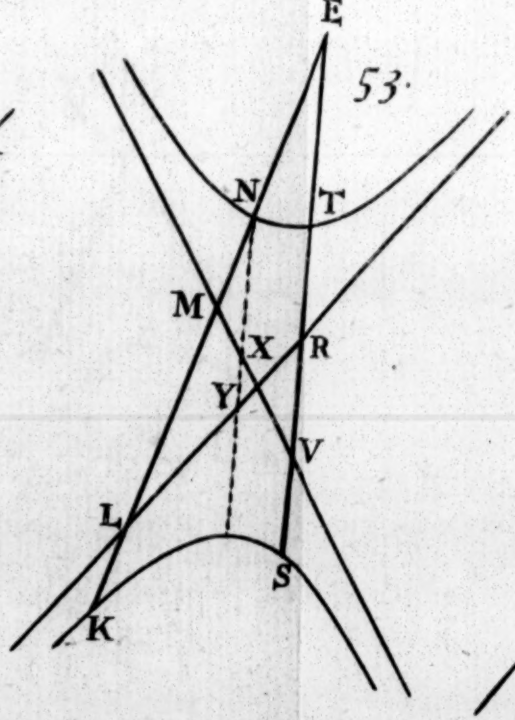

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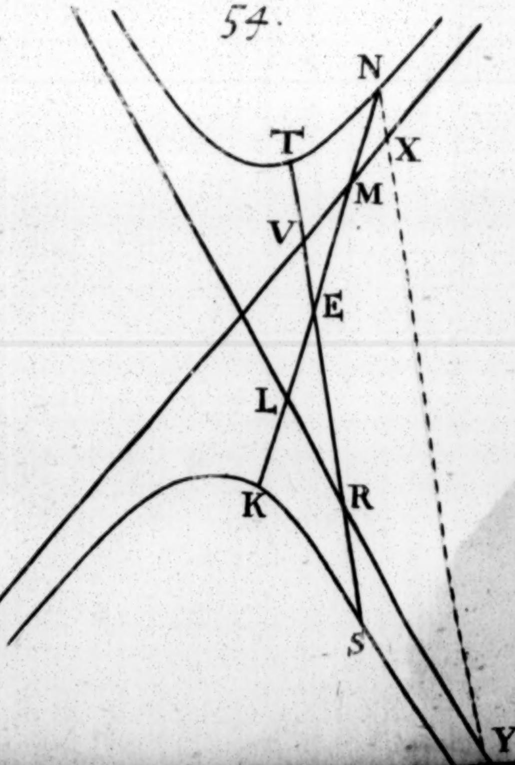

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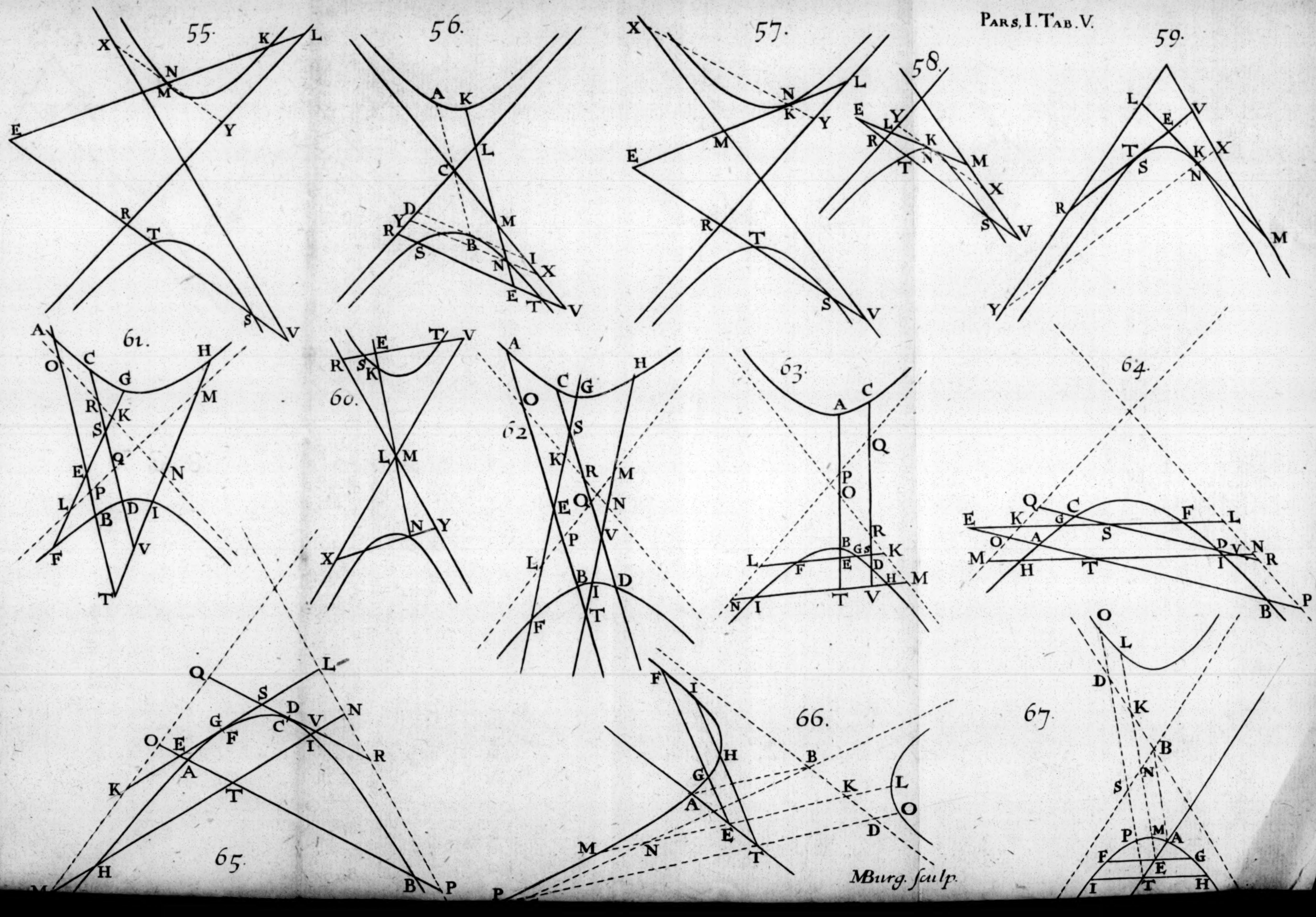
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A C Q P O R B G S K L F E D H M N I T V
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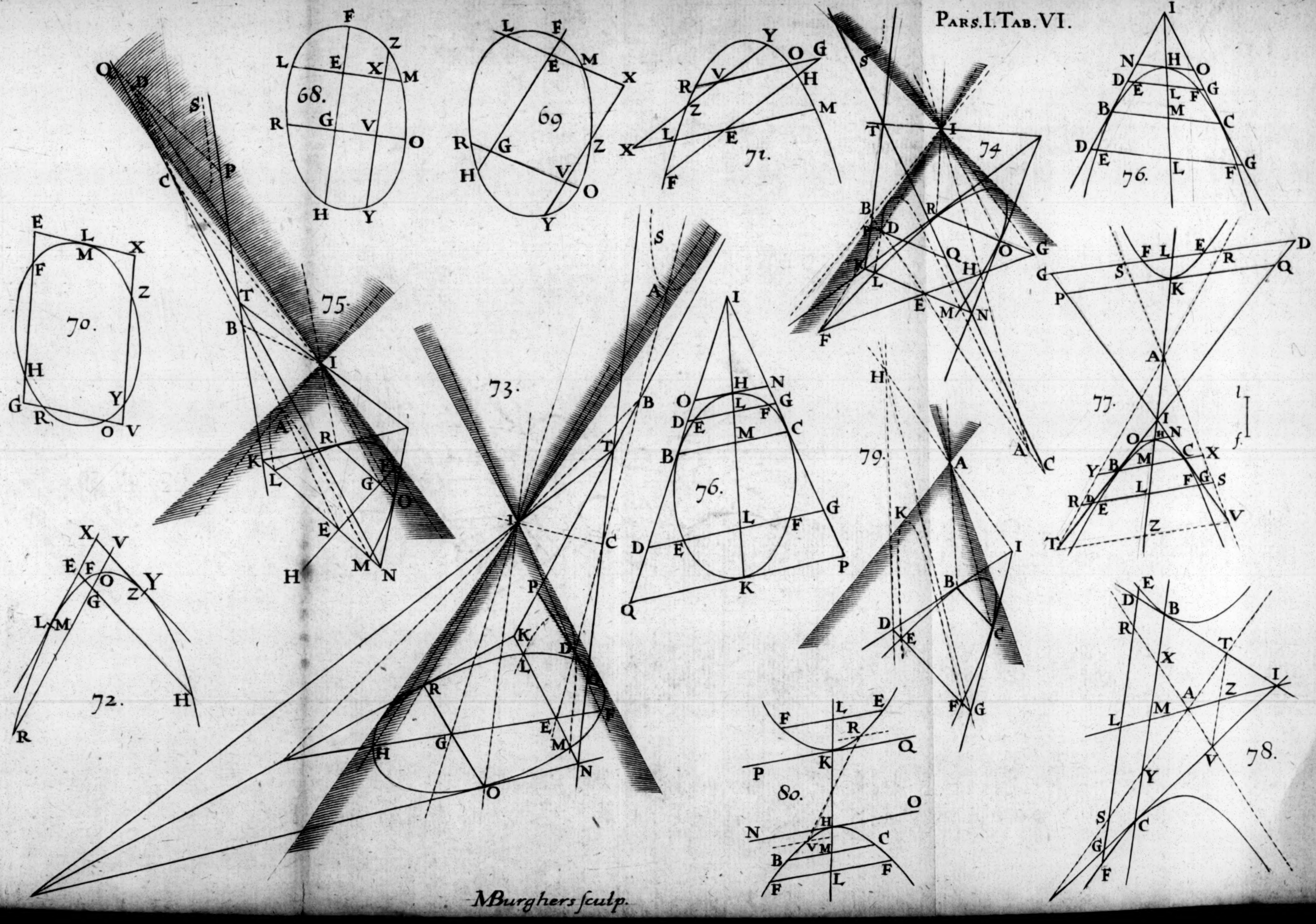

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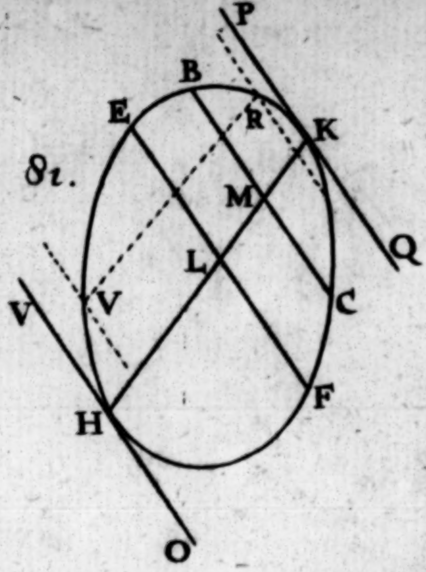

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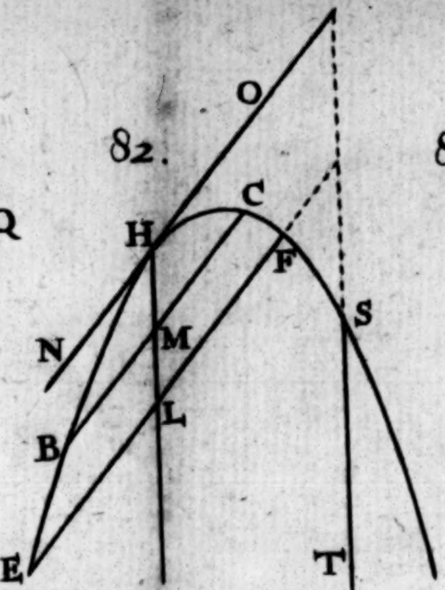

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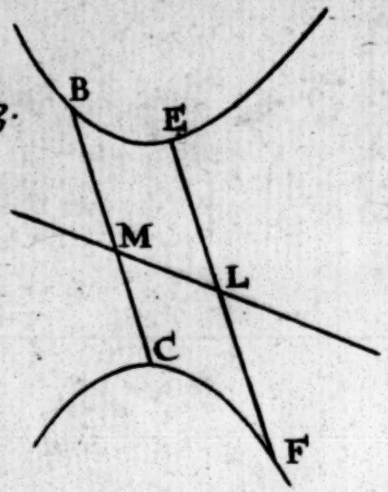

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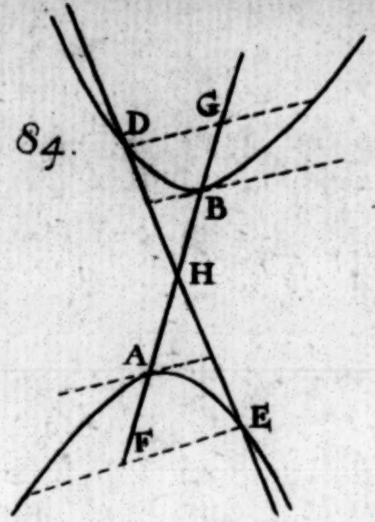

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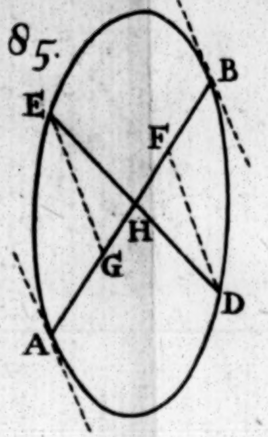

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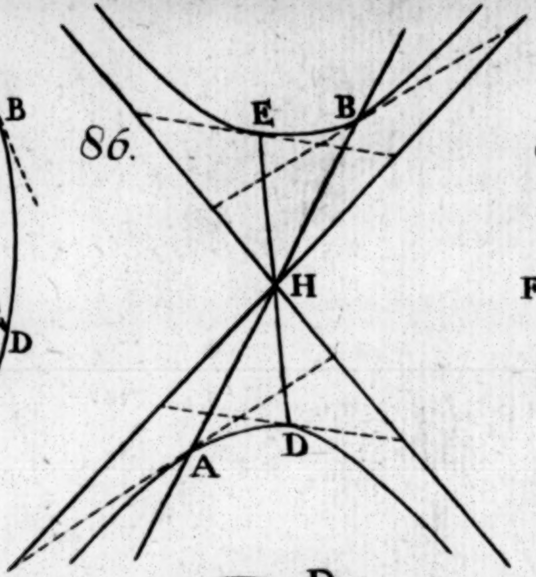

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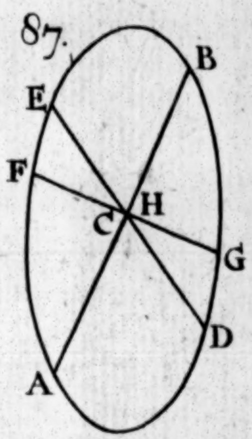

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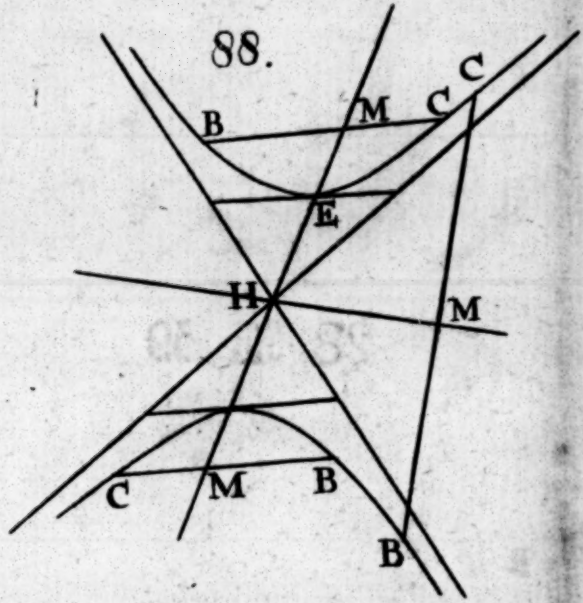

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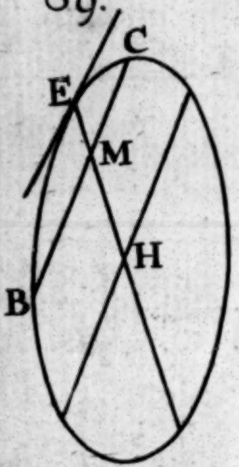

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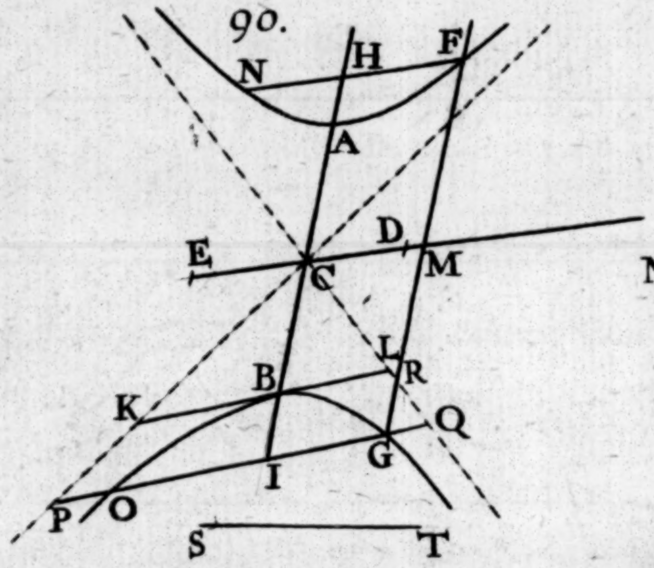

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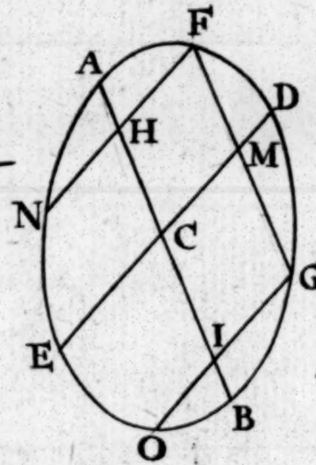

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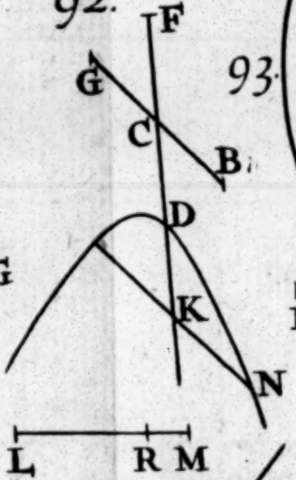

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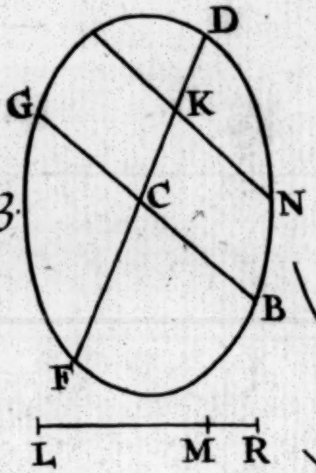

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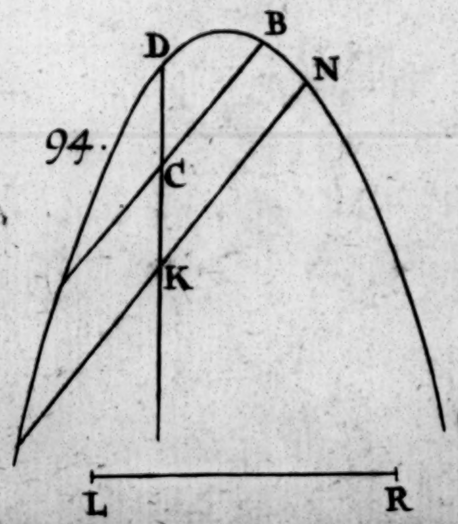

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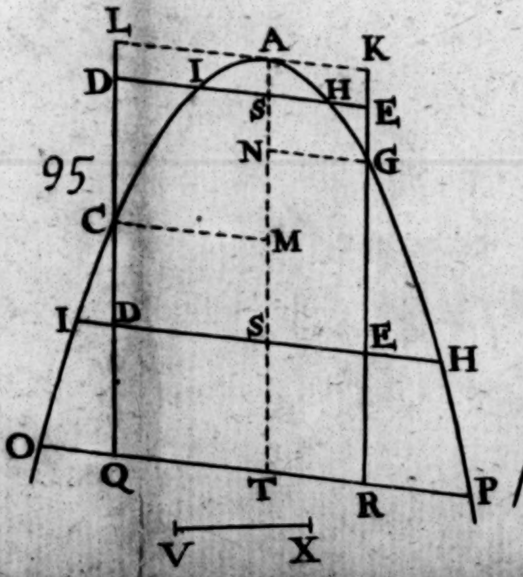

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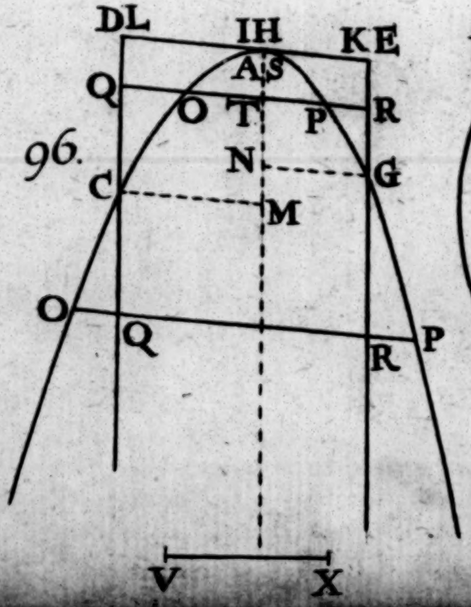

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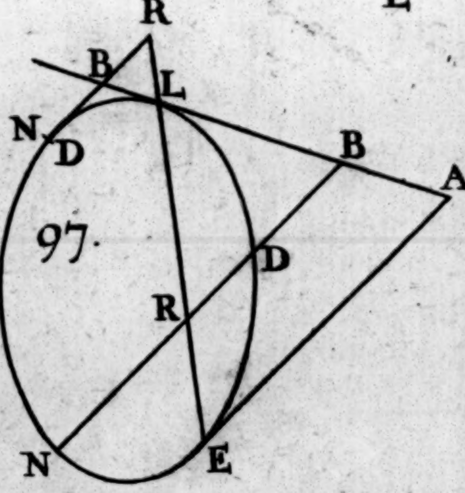

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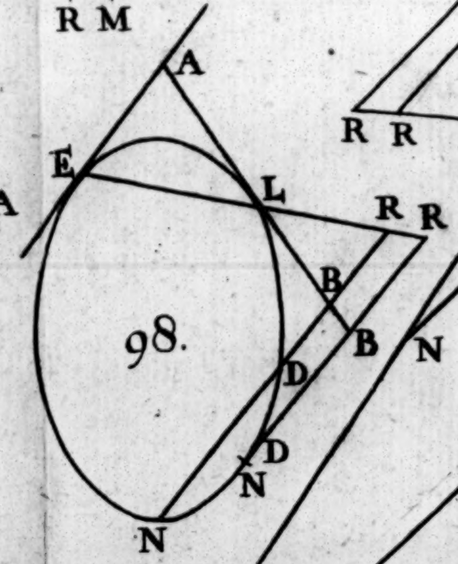

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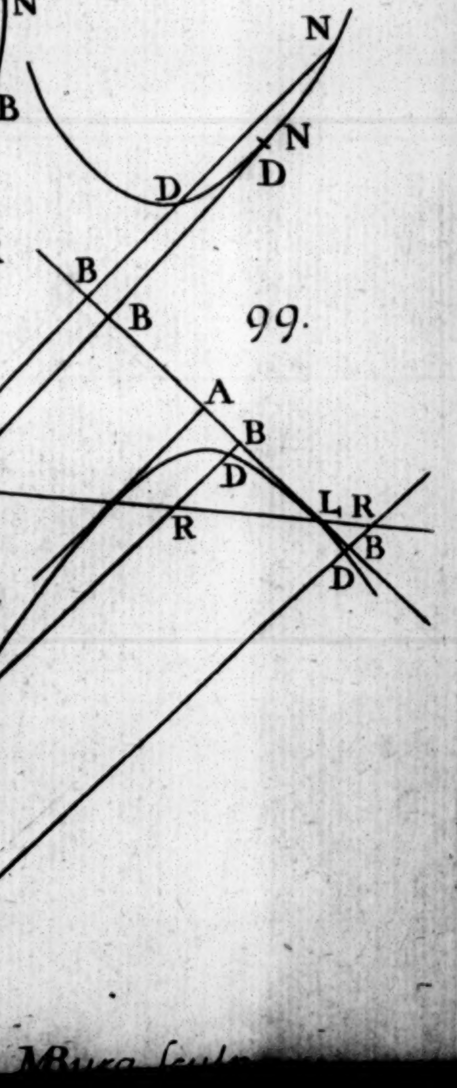

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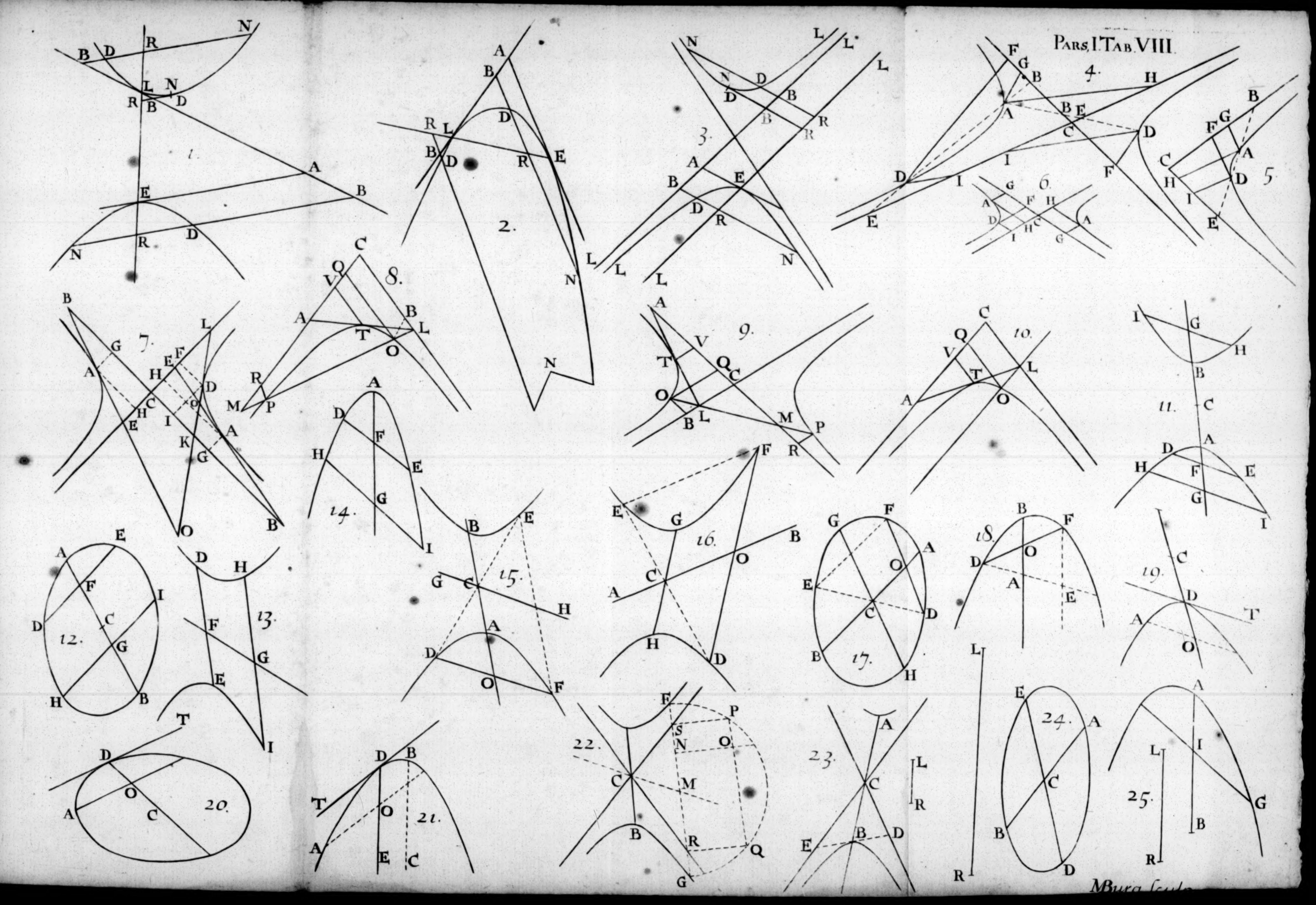
PARS. I. TAB. VIII.



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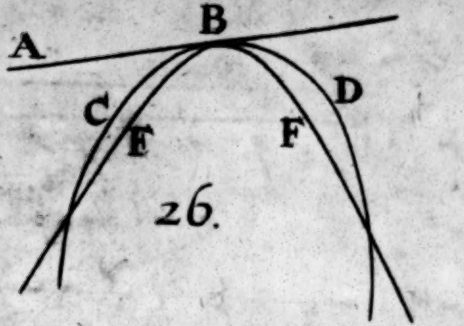

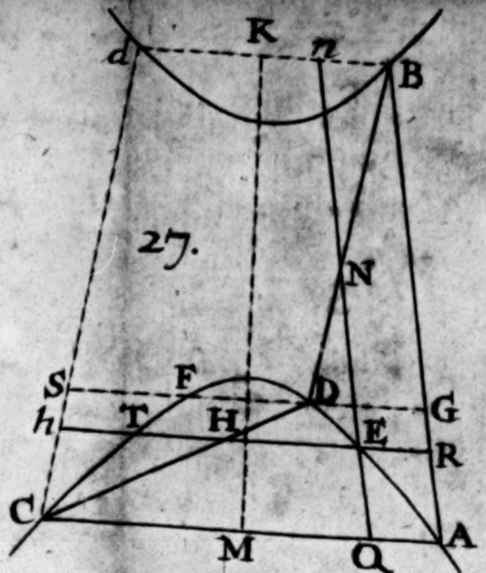

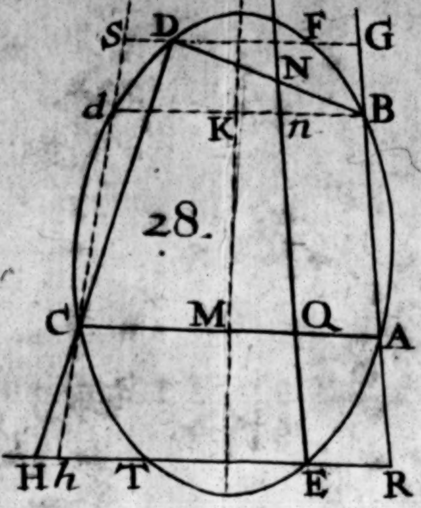

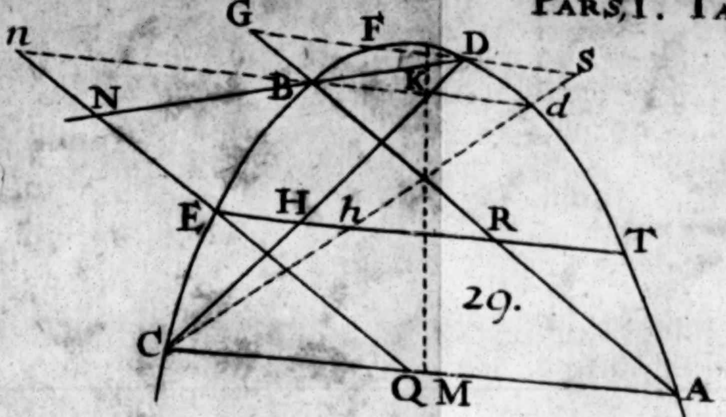

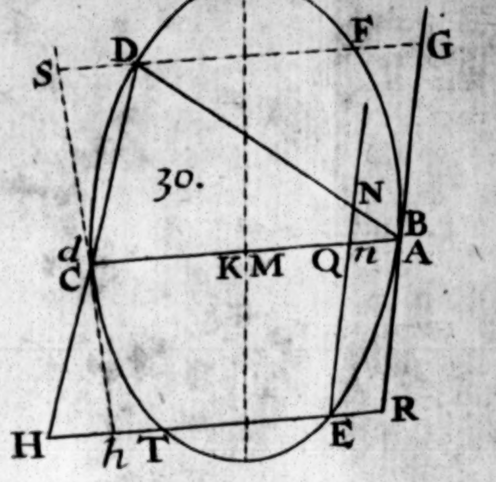

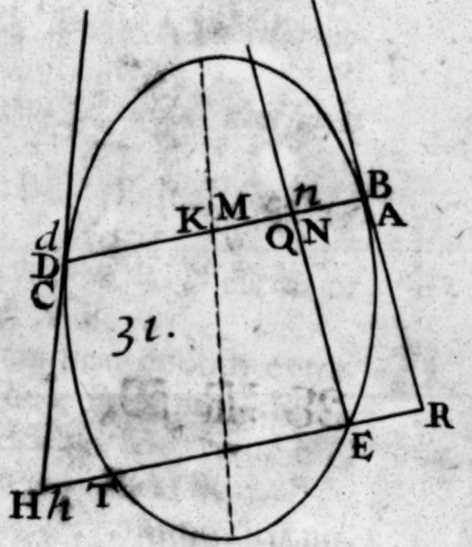

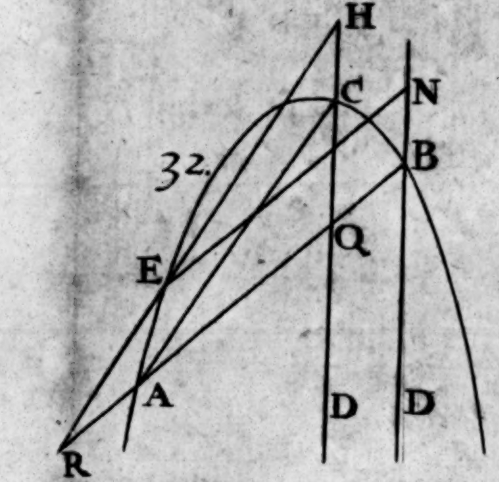

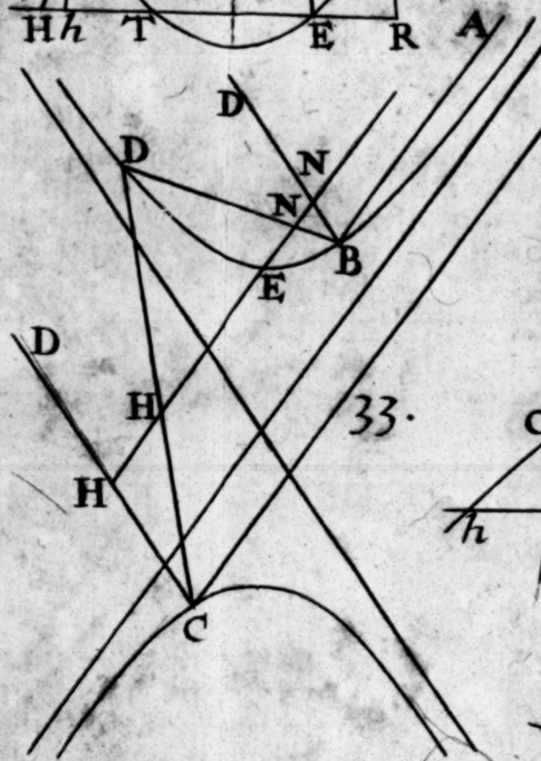

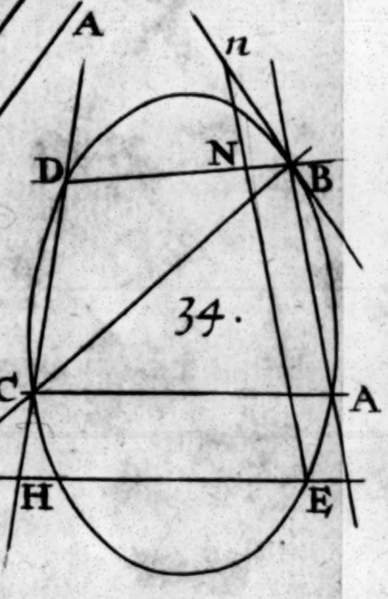

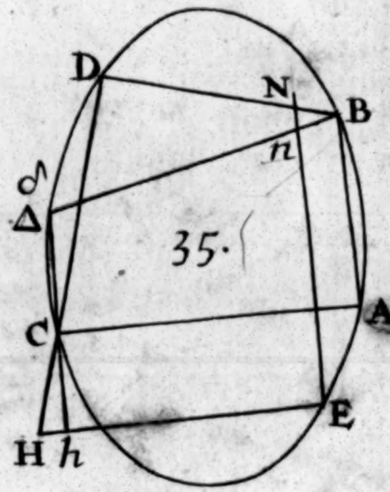

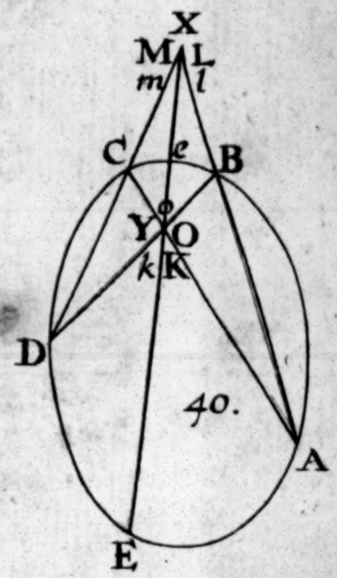

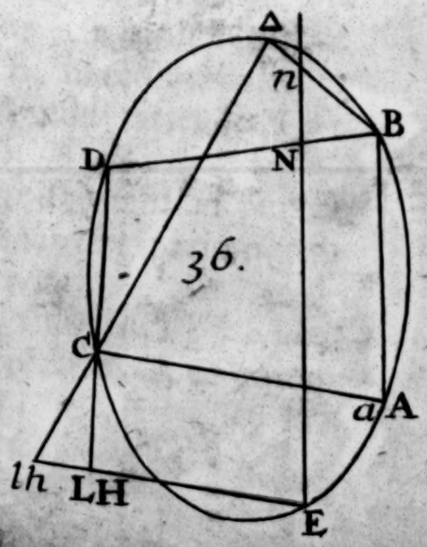

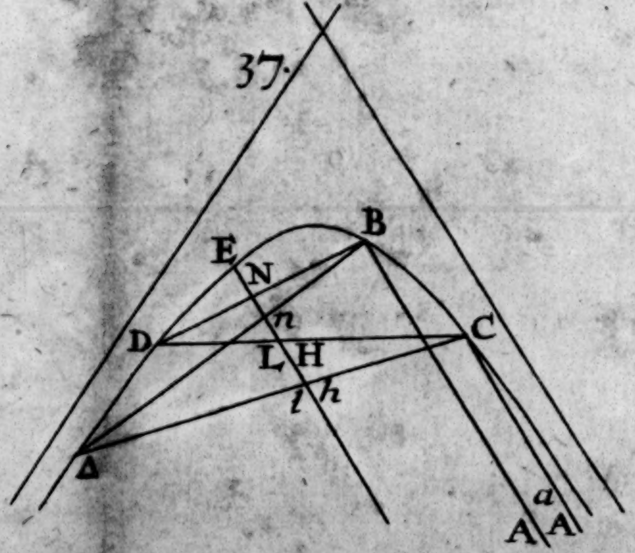

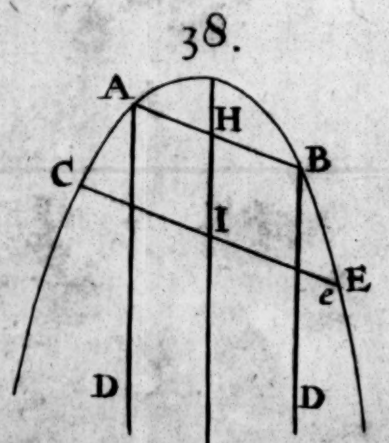

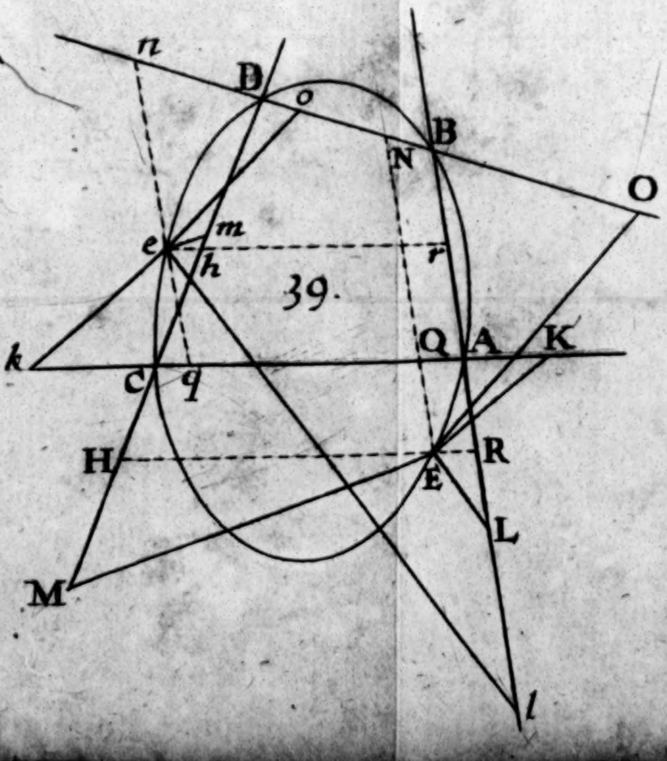

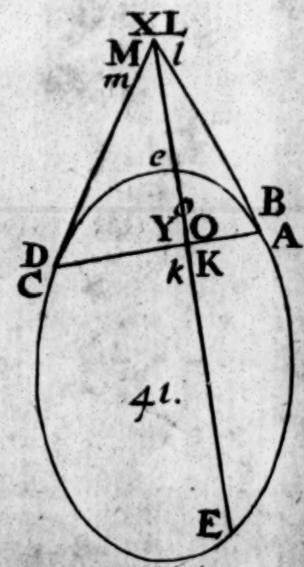

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PARS II.

Definitiones.

SI Recta A D ita dividatur in B, C, ut ſit tota A D 1.
ad utramvis partem extremam C D, ut reliqua extrema A B ad mediam B C, Recta A D *Harmonicè diviſa* dicitur.

Et Puncta A, B, C, D dicuntur *Puncta diviſionis Harmonicæ*, Vel *Harmonicalia*.

Coroll. 1. Ob A D : D C : : A B : B C. Erit alternando A D : A B : : D C : B C.

Coroll. 2. Utravis extremarum A B vel C D eſt major mediâ B C. nam $AD \sqsubset CD$, vel A B.

Coroll. 3. Datis diviſionis harmonicæ duobus extre- 2.
mis punctis E, H, & mediorum uno F, invenietur alterum G. Nempe dividendo F H in G in ratione E H ad E F ; Vel E F in G in ratione E H ad H F.

Coroll. 4. Sint E, F, G, H puncta diviſionis harmo- 3.
nicæ, ſitque I F utriuſvis extremæ partis E F exceſſus ſupra mediam F G ; Propter E F : F G : : E H : G H erit divid.

$$\left.\begin{matrix}EF-FG\\ IF\end{matrix}\right\}:FG::\left\{\begin{matrix}EH-HG\\ EG\end{matrix}\right\}:HG.$$

Coroll. 5. Unde, datis diviſionis harmonicæ mediis 3.
punctis F G, & extremorum uno E, invenitur alterum H. Factâ nempe E I = F G ; Deinde faciendo I F : F G : : E G : G H.

Coroll. 6. In caſu *Coroll.* 3^{i}. liquet duo tantùm puncta propoſito ſatisfacere ; ex utrâque ſcilicet parte puncti F unum ; In caſu *Coroll.* 5^{i}. unicum.

Coroll. 7. Poſitis ut in *Coroll.* 4. Cum ſit I F : F G : : 3. 4. 5.
E G : H G ; quo minor eſt I F reſpectu F G, eo minor 6.

G erit

erit EG reſpectu HG: ſi itaque ſit $EF = FG$, hoc eſt, ſi IF nullius ſit magnitudinis, erit HG infinita. Si ſit EG finita, HG vero infinita, erit IF nullius magnitudinis; hoc eſt, erit $EF = FG$. Cujus rei occurrent exempla in ſequentibus.

Lemma 1.

7. *Rectâ* AD *Harmonice diviſâ in* B, C, *& utrâvis extremâ parte* AB *ſimul cum mediâ* BC, *i. e.* AC *biſectâ in* M, *Dico*

$MB : MC : MD$ ∺

Productâ DA, fiat $Ad = CD$;

1. Ex hyp. $AB : BC :: AD : DC$
2. Comp. $\left.\begin{matrix}AB+BC\\AC\end{matrix}\right\} : BC :: \left\{\begin{matrix}AD+DC\\Dd\end{matrix}\right\} : DC,$

Et dimidiatis antecedentibus.

3. $MC : BC :: MD : DC;$
4. Divid. $\left.\begin{matrix}MC-BC\\MB\end{matrix}\right\} : MC :: \left\{\begin{matrix}MD-DC\\MC\end{matrix}\right\} : MD$

Lemma 2.

7. *Iiſdem poſitis, Dico* $BC : BD :: BM : BA$.

Nam ex 3. proport. ſuprà,

$BC : DC :: MC : MD$, & per 4^{m}. proport.

$MB : MC :: MC : MD$ unde

$BC : DC :: MB : \left\{\begin{matrix}MC\\MA\end{matrix}\right\}$ Et comp.

5. $BC : \left\{\begin{matrix}BC+DC\\BD\end{matrix}\right\} :: MB : \left\{\begin{matrix}MB+MA\\BA\end{matrix}\right\}$

Lemma 3.

7. *Iiſdem poſitis, Dico* $DC : DB :: DM : DA$.

Nam ex 3. proport. ſupra,

$MD : MC :: DC : BC$, & ex 1^{a}. proport.

$DC : BC :: AD : AB$; Unde

6. MD : MC :: AD : AB, Et divid.

$$\left.\begin{matrix}MD - MC\\ CD\end{matrix}\right\} : MD :: \left\{\begin{matrix}AD - AB\\ BD\end{matrix}\right\} : AD;$$

7. Unde altern. CD : BD :: MD : AD.

Lemma 4.

Iiſdem poſitis, *Dico* DC : DB :: AM : AB. 7.
Nam ex 5. proport.
BD : BC :: BA : MB. Et div.

$$\left.\begin{matrix}BD - BC\\ DC\end{matrix}\right\} : BD :: \left\{\begin{matrix}BA - MB\\ AM\end{matrix}\right\} : BA. \qquad (8.)$$

Lemma 5.

Iiſdem poſitis, *Dico* AD : BD :: AM : BC. 7.
Nam ex proport. 1.
BA : AD :: BC : DC. & ex proport. 8.
BA : BD :: AM : DC. Unde
AD : BD :: AM : BC.

Def. In rectâ quâvis AD ſi per diviſionis harmonicæ puncta A,B,C,D agantur quatuor rectæ AE, BE, CE, DE in puncto quovis E extra rectam AD concurrentes, vel inter ſe parallelæ; Eædem *Harmonicales* appellentur. 8. 9.

Coroll. Recta quævis ipſi AD parallela harmonicalibus in *a*, *b*, *c*, *d* occurrens harmonicè dividitur. Nam (ob parallelas rectas) ſecatur in eadem ratione quâ AD.

Lemma 6.

Iiſdem poſitis, *ſit recta quævis* FH *cuivis harmonicalium* ED *vel* EC *parallela, tribus reliquis occurrens in* F, G, H, *Dico* FG, *biſectam in* G. 10. 11.

Per G ductâ IKGL ipſi AD parallelâ, ſimilia ſunt triang. IKE, GKF, & ILE, GLH. Per *Coroll.* ad præced. def. IL : LG :: IK : KG :: (per ſim. triang.) IE : FG :: IE : GH. Unde FG = GH.

Lemma 7.

12. *Et converſim, Rectâ quâvis* F G H *biſectâ in* G, *&* *per quodvis punctum* E *extra rectam* F H *actis rectis* F E, G E, H E, *&* E D *ipſi* F H *parallelâ, Dico rectas* F E, G E, H E, D E, *eſſe Harmonicales.*

Per G ducatur K G L I occurrens quatuor rectis in K, G, L, I, quod fieri poſſe manifeſtum eſt; Erunt (ob F H || E D) ſimilia triangula I K E, G K F, uti & I L E, G L H, unde

$$IE : GH :: IL : LG, \text{ Et}$$

$$IE : \left\{\begin{matrix} FG \\ GH \end{matrix}\right\} :: IK : KG; \text{ Ergo ex æquo}$$

$$IL : LG :: IK : KG.$$

Hoc eſt, puncta K, G, L, I ſunt harmonicalia; & rectæ F E, G E, H E, D E harmonicales.

Lemma 8.

13. *Si quatuor harmonicales à recta quâvis utcunque ſecentur in punctis* K, G, L, I, *Dico rectam* K I *harmonicè dividi.*

Si rectæ harmonicales concurrant in E, ſumptis duobus quibuſvis alternis punctis I, G, per alterum G ducatur H G F harmonicali I E, quæ per alterum I tranſit, parallela, tribus reliquis occurrens in H, G, F; Erit (per lemma 6.) F G = G H, & triangula I K E, G K F, uti & I L E, G L H ſimilia; Unde

$$IE : GH :: IL : LG, \text{ Et}$$

$$IE \left\{\begin{matrix} FG \\ GH \end{matrix}\right\} :: IK : KG; \text{ Ergo ex æquo.}$$

$$IL : LG :: IK : KG.$$

Si harmonicales ſint parallelæ, res per ſe manifeſta eſt.

Lemma 9.

14. 15. 16. &c. *Si duæ rectæ* A B C D, A F G H *harmonicè diviſæ habeant quodlibet diviſionum punctum* A *commune, cæteraque*

teraque diviſionum puncta rectis B F, C G, D H *ordinatim conjungantur, i. e. primum unius cum primo alterius, ſecundum cum ſecundo, tertium cum tertio; à communi ſcilicet puncto verſus utrumvis extremum progrediendo, & per alterum extremum (ſi opus) redeundo; vel, quod idem eſt, ſecundum unius cum ſecundo alterius conjungatur, cætera pro libitu; Dico rectas* B F, C G, D H *vel coire omnes in communi puncto* E, *vel eſſe omnes inter ſe parallelas.*

Primò ſi earum binæ quælibet *ex. gr.* H D, C G 14. 15.
coeant in E. Juncta B E ſecet A F G H in I, aut (ſi 16. 17.
fieri poteſt) ſit ei parallela, & connectatur A E: Si 18. 19.
B E ſecet A F G H in I, ob diviſionem harmonicam erunt E A, E B I, E C G, E D H harmonicales, & puncta A, I, G, H harmonicalia, uti etiam ex hypotheſi ſunt A, F, G, H; ſecundo vero unius rectæ puncto cum ſecundo alterius connexo, puncta F, I ſemper cadent ad eaſdem partes reſpectu reliquorum;
Nam ſi punctum F ſit medium (ſcilicet inter A, G,) 14. 15.
hoc manifeſtum eſt, cum A G angulum A E G ſubtendat, 16. 19.
quam ducta E B I dividit; coincident ergo puncta I, F, cùm (per *coroll.* 6. ad def. harmon. div.) inter A, G non ſit aliud medium ab F diverſum. Sin F 17. 18.
extremum fuerit, idem nihilominùs eveniet; non enim cadit I inter A, G medium, cum E B extra angulum A E G cadat; cumque ex punctis A, G, H duo ſint media & unum extremum, non erunt (per idem *coroll.* 6.) duo adhuc extrema I, F à ſe invicem diverſa. Coincidunt ergo I, F. *i. e.* F B tranſit per E.

Si E B dicatur parallela A F G H, erit A H = H G contra hypotheſin.

Si duæ aliquæ conjungentium G C, H D parallelæ 20. 21.
ſint, Ob ſimilia triangula erit

$$CA : AD :: GA : AH \text{ \& comp. vel div.}$$

$$\left.\begin{matrix}AD \pm CA\\ CD\end{matrix}\right\} : AD :: \left\{\begin{matrix}AH \pm GA\\ GH\end{matrix}\right\} : AH ::$$

(ob div. harm.) C B : A B : : G F : F A unde triangula A B F, ACG ſimilia ſunt, hoc eſt, BF, CG, DH parallelæ.

Si puncta harmonicalia aliâ lege conjungantur, Duæ ſaltem conjungentium concurrent inter puncta connexa, duæ vero non; Unde non erit communis omnium concurſus.

Lemma 10.

22. 23. *Si recta harmonicè diviſa* ABCD, *& alia bifariam*
24. 25. *diviſa* BFG, *habeant quodlibet diviſionum punctum*
26. 27. *commune* B, *& duo reliqua bifariam diviſæ puncta cum totidem alterius rectis* AF, CG *conjungantur, & per reſiduum harmonicè diviſæ punctum* D *agatur* DH *ipſi* BFG *parallela*; *Ita tamen ut cum punctum* B *fuerit bifariam diviſæ extremum, ſecundum unius cum ſecundo alterius conjungatur, ſin* B *medium fuerit bifariam diviſæ punctum, punctum* D *per quod parallela recta ducitur ſit à communi* B *ſecundum*; *Cætera pro libitu. Dico rectas* AF, CG, DH *coire omnes in communi quodam puncto* E.

Ob rectarum, quarum puncta conjunguntur, partes unius quidem æquales, alterius inæquales, liquet AF, CG concurrere. Porro manifeſtum eſt, AF, CG non eſſe ipſi BFG parallelas, proindeque utramque huic parallelæ DH occurrere.

Sit ergo ipſarum AF, CG occurſus E; Per E ductâ EI parallelâ BFG ſecante ABCD in I, Ob BFG bifariam diviſam, & huic parallelam EI, (connexâ EB,) erunt EB, EA, EC, EI harmonicales, & puncta B, A, C, I harmonicalia, uti ſunt ex hypotheſi B, A, C, D, unde oſtendetur (ut in priore lemmate) puncta D, I coincidere, hoc eſt, rectam DH tranſire per E.

Schol. Lemma hoc eſt (propriè) præcedentis caſus particularis; nec puncta conjungendi lex hîc poſita ab illâ diverſa eſt; cùm (per *coroll.* 7. ad *def.* præced.) diviſio bifariam ſit diviſio harmonica, cujus punctum unum, ad utramvis partem, in infinitum abit; ideoque recta ab alterius rectæ puncto aliquo ducta bifariam diviſæ parallela, pro puncta conjungente habenda.

Propoſitio I. Theorema I.

Si ſectionem quamvis conicam, vel ſectiones oppoſitas, 28, 29.
contingant binæ rectæ A D, A G *concurrentes in* A; 30. 31.
per punctum verò concurſûs A *agatur recta* A L K I, 32.
ſectioni, vel utrique ſectionum oppoſitarum occurrens
in L, I, *&* *conjungenti tactus* D G (*in opp. ſect.*
productæ) *in* K; *Dico eandem harmonicè dividi à*
punctis A, L, K, I; *i. e.* A L : A I :: K L : K I.

Hæc propoſitio eſt caſus corollarii propoſitionis 36. p. 1. Ob inſignem verò & frequentem ejus in ſequentibus uſum, viſum eſt eam denuò propoſitam peculiari demonſtratione munire, ſingulis caſibus ſingulis figuris expreſſis.

Per I, L actæ E I M H, B L N F ipſi D G parallelæ occurrant contingentibus in E, H, & B, F, ſectioni vero, vel ſectioni oppoſitæ, vel utrique ſectionum oppoſitarum in M, N; Erit (per Prop. 19. Part. 1.) M H = E I & N F = B L *i. e.* M E = H I & N B = F L. Et (per Prop. 17, vel 18. Part. 1.) erit

$$\left.\begin{matrix} LB \times NB \\ LB \times LF \end{matrix}\right\} : \left\{\begin{matrix} EI \times EM \\ EI \times HI \end{matrix}\right\} :: DBq : DEq :: ($$

ob rectas parallelas) K L q : K I q. Rurſus ob ſim triang.

L B : E I :: A L : A I &
L F : H I :: A L : A I. Ductiſque &c.

L B × L F : E I × H I :: A L q : A I q :: (priùs) K L q : K I q. Ergo A L : A I :: K L : K I.

Coroll. 1. In hyperbola & Ellipſi, Si recta contingens 33. 34.
ſectionem in quolibet puncto D, cuivis diametro I L (ſi opus productæ) occurrat in A, & à tactu D ad eandem diametrum ordinetur recta D K; Vel ſi ad diametrum quamvis I L ordinetur recta D K, in cujus extremo D recta D A ſectionem contingens occurrit dia-

metro in A; Eadem diameter in punctis L, K, I, A harmonicè ſecabitur, *i. e.* erit A L : A I : : K L : K I. Nam productâ D K donec ſectioni denuò occurrat in G, Contingens in puncto G (per *Coroll.* 17. prop. 20. part. 1.) occurret contingenti A D ſuper diametro I L in A. Unde hic caſus non differt ab illo hujus propoſitionis.

33. 34. *Coroll.* 2. Unde (cum centrum hyperbolæ vel Ellipſeos C biſecet L I) erit per Lemma 1. hujus partis.

C K : C I : C A ∺

Et per Lemma 2^{m}. in Ellipſi per 3^{m}. in Hyperbola.

L K : C K : : A K : I K.

Et per Lemma 2. in Hyper. per 3. in Ellipſi.

L A : C A : : K A : I A

Et per Lemma 4. in Hyper. per 5. in Ellipſi.

A L : A K : : C L : I K.

Et per Lemma 4. in Ellipſi per 5. in Hyperb.

L K : L C : : K A : I A.

35. *Coroll.* 3. Si contingens Parabolam in quolibet puncto D cuilibet diametro I L occurrat in A, & à D ordinetur ad hanc diametrum recta D K, erit A I = I K. Nam productâ K D in G, Contingentes in D, G, ut ſupra in *Coroll.* 1. coeunt ſuper diametro in A, Recta verò A I K L in quovis alio ſitu A *i k l* ſectioni occurrit in binis punctis *i*, *l*, & (per hanc prop. 1.) harmonice dividitur à punctis A, *i*, *k*, *l*; Jam rectâ A *i k l* ſitum A I K L obtinente, abit punctum L in infinitum, unde (per *Coroll.* 6. ad def. div. harm.) A I = I K. Hanc vero parabolæ proprietatem peculiari Theoremate mox dignabimur.

36. 37. *Coroll.* 4. In hyperbolâ vel opp. ſect. ſi recta A I K L ſit utrivis aſymptoto parallela, erit A I = I K. Na in quovis alio ſitu A *i k l* ex una parte, erit A *i*, in di viſione harmonicâ, pars extrema, & *i k* media, i. A *i* ⊏ *i k*; ex altera parte erit *i k* extrema & A *i* medi i. e. *i k* ⊏ A *i*; proindeque in ſitu A I K L erit A = I K. Patet etiam eodem modo quo in corollari præcedenti.

Coro

roll. 5. In hyperbolâ & oppoſitis ſectionibus, ſi 38. 39.
ngentium altera *ex. gr.* A D (tactu D migrante in 40.
itum) degeneret in aſymptoton , conjungens ta-
G D fit recta eidem aſymptoto parallela, idemque
tat quod tactus conjungens in caſibus hujus prop. 1.
roll. 4. nimirum ſi A I K L occurrat ſectioni, vel 38. 39.
onibus oppoſitis, in binis punctis, erit ut in caſu
s prop. A L : A I : : K L : K I. Si vero A I K L 40.
lteri aſymptoto C E parallela, erit ut in caſu *Co-*
4. A I = I K. Nam quod in alio quovis ſitu rectæ
ex utrâvis parte obtinet, in ſitu intermedio obti-
t.

oroll. 6. Et ut proprietates, quantumvis ſpecie di- 41.
ctas, aliquali affinitatis vinculo conjunctas eſſe, & in
mutuò tranſire innoteſcat; Si in hyperbolâ, vel
opp. contactuum D, G uterque in infinitum abeat,
ingens utraque fit aſymptotos, harumque occur-
A centro coincidit; punctoque K cum tactus con-
ente in infinitum migrante, fit (per idem *Coroll.*)
= A I; Eſt vero in hoc caſu L I diameter, atque
etiam L A = A I.

n Ellipſi vero vel ſect. opp. Si contingentium oc- 42. 43.
ſus A migret in infinitum, erit (per idem *Coroll.*)
= K I; ſed & propter punctum A infinitè diſtans
nt D A, L A, G A invicem parallelæ, unde erit D G
meter, & L I ad hanc ordinata, atque hinc etiam
= K I.

Prop. II. Theor. II.

parabolam, cujus diameter quælibet A K, *& ejus* 44.
ertex I, *recta* A D *ubivis in* D *contingens*, *eidem*
iametro occurrat in A, *& à tactu* D *ordinetur ad*
iametrum recta D K; *Dico* A I = I K.

Per D ductâ diametro D M, ad hanc ordinetur I G O,
æ erit propterea parallela A D, & biſecta in G; Ab
ad diametrum A K ordinetur O N quæ erit ideo pa-
lela D K, eruntque triangula K D A, N O I ſimilia;

metro in A; Eadem diameter in punctis L, K, I,
harmonicè ſecabitur, *i. e.* erit A L : A I : : K L : K
Nam productâ D K donec ſectioni denuò occurrat
G, Contingens in puncto G (per *Coroll.* 17. prop. 2
part. 1.) occurret contingenti A D ſuper diametro I
in A. Unde hic caſus non differt ab illo hujus pro
ſitionis.

33. 34. *Coroll.* 2. Unde (cum centrum hyperbolæ vel I
lipſeos C biſecet L I) erit per Lemma 1. hujus partis

C K : C I : C A ÷

Et per Lemma 2^{m}. in Ellipſi per 3^{m}. in Hyperbo

L K : C K : : A K : I K.

Et per Lemma 2. in Hyper. per 3. in Ellipſi.

L A : C A : : K A : I A

Et per Lemma 4. in Hyper. per 5. in Ellipſi.

A L : A K : : C L : I K.

Et per Lemma 4. in Ellipſi per 5. in Hyperb.

L K : L C : : K A : I A.

35. *Coroll.* 3. Si contingens Parabolam in quolibet pun
D cuilibet diametro I L occurrat in A, & à D ordi
tur ad hanc diametrum recta D K, erit A I = I
Nam productâ K D in G, Contingentes in D, G, ut
pra in *Coroll.* 1. coeunt ſuper diametro in A, Recta v
A I K L in quovis alio ſitu A *i k l* ſectioni occurri
binis punctis *i*, *l*, & (per hanc prop. 1.) harmonice
viditur à punctis A, *i*, *k*, *l*; Jam rectâ A *i k l* ſi
A I K L obtinente, abit punctum L in infinitum, u
(per *Coroll.* 6. ad def. div. harm.) A I = I K. H
vero parabolæ proprietatem peculiari Theoremate m
dignabimur.

36. 37. *Coroll.* 4. In hyperbolâ vel opp. ſect. ſi recta A I K L
ſit utrivis aſymptoto parallela, erit A I = I K. N
in quovis alio ſitu A *i k l* ex una parte, erit A *i*, in
viſione harmonicâ, pars extrema, & *i k* media,
A *i* ⊏ *i k*; ex altera parte erit *i k* extrema & A *i* me
i. e. *i k* ⊏ A *i*; proindeque in ſitu A I K L erit
= I K. Patet etiam eodem modo quo in corolla
præcedenti.

Cor

Coroll. 5. In hyperbolâ & oppoſitis ſectionibus, ſi 38. 39.
contingentium altera *ex. gr.* A D (tactu D migrante in 40.
infinitum) degeneret in aſymptoton, conjungens tactus G D fit recta eidem aſymptoto parallela, idemque
præſtat quod tactus conjungens in caſibus hujus prop. 1.
Coroll. 4. nimirum ſi A I K L occurrat ſectioni, vel 38. 39.
ſectionibus oppoſitis, in binis punctis, erit ut in caſu
hujus prop. A L : A I : : K L : K I. Si vero A I K L 40.
ſit alteri aſymptoto C E parallela, erit ut in caſu *Coroll.* 4. A I = I K. Nam quod in alio quovis ſitu rectæ
G D ex utrâvis parte obtinet, in ſitu intermedio obtinebit.

Coroll. 6. Et ut proprietates, quantumvis ſpecie di- 41.
ſtinctas, aliquali affinitatis vinculo conjunctas eſſe, & in ſe mutuò tranſire innoteſcat; Si in hyperbolâ, vel ſect. opp. contactuum D, G uterque in infinitum abeat, contingens utraque fit aſymptotos, harumque occurſus A centro coincidit; punctoque K cum tactus contingente in infinitum migrante, fit (per idem *Coroll.*) L A = A I; Eſt vero in hoc caſu L I diameter, atque inde etiam L A = A I.

In Ellipſi vero vel ſect. opp. Si contingentium oc- 42. 43.
curſus A migret in infinitum, erit (per idem *Coroll.*) L K = K I; ſed & propter punctum A infinitè diſtans erunt D A, L A, G A invicem parallelæ, unde erit D G diameter, & L I ad hanc ordinata, atque hinc etiam L K = K I.

Prop. II. Theor. II.

Si parabolam, cujus diameter quælibet A K, *&* *ejus* 44.
vertex I, *recta* A D *ubivis in* D *contingens, eidem diametro occurrat in* A, *&* *à tactu* D *ordinetur ad diametrum recta* D K; *Dico* A I = I K.

Per D ductâ diametro D M, ad hanc ordinetur I G O, quæ erit propterea parallela A D, & biſecta in G; Ab O ad diametrum A K ordinetur O N quæ erit ideo parallela D K, eruntque triangula K D A, N O I ſimilia;

Ob $AD = IG$ erit $IO = 2\,AD$; ideoque ob ſimil. triang. $NI = 2\,AK$, & $NO = 2\,KD$: Sed per *Coroll.* 4. prop. 20. p. 1. $NOq = 4\,KDq : KDq :: NI : KI$, *i. e.* $NI = 4\,KI$. Ergo $4\,KI = 2\,AK$ *i. e.* $2\,KI = AK$ *i. e.* $KI = AI$.

In figuris prop. 1. & Corollariorum ejus 1, 2, 3, 4, & 5.

Coroll. ad hanc & præced. prop. Si ex eodem puncto A recta AD ſectionem vel unam è ſect. opp. contingat, altera ALI ſectionem vel ſectiones oppoſitas ſecet in duobus punctis L, I, vel in unico I ſi ſit Hyperbolæ aſymptoto parallela, vel Parabolæ diameter; Datiſque tribus A, L, I inveniatur quartum diviſionis harmonicæ punctum K, quod ſit medium inter L & I ſi puncta L, I ſint ad eandem ſectionem, extremum verò ſi ſint ad oppoſitas; Vel (ſi AI ſit aſymptoto parallela, vel parabolæ diameter) ſi fiat $IK = AI$; & connectatur KD occurrens denuò ſectioni, vel ſectioni opp. in G; Vel (ſi AD ſit aſymptotos,) ſi ducatur KG parallela AD ſecans ſectionem in G: Erit connexa AG contingens. Nam datis tribus A, L, I non erit aliud har. div. punctum extremum, vel inter data L, I medium, præter K; Nec aliud punctum diviſionis bifariam ad partes ſectionis præter K; unde ex prop. 1. & 2. ſatis liquet propoſitum.

Prop. III. Theor. III.

45. 46. 47. 48. *Si ſectionem quamvis conicam, vel ſectiones oppoſitas contingant binæ rectæ* AF, AG, *concurrentes in* A, *& per* A *ducatur* AV *tactus conjungenti* FG *parallela; Sumptoque in* AV *quolibet puncto* V, *per* V *& medium punctum* O *tactus conjungentis ducatur* VO *occurrens ſectioni, vel ſectionibus oppoſitis in binis punctis* T, L; *Dico eandem harmonicè dividi à punctis* V, T, O, L.

Connexa AL ſectioni occurrat in Q, & rectæ FG in R, & ducatur QP parallela FG, eidem ſectioni denuò, vel ſectioni oppoſitæ occurrens in P; erit connexa A O diameter, biſecans QP in S. Propter A L (per præced.) harmonicè diviſam, erunt O L, O R, O Q, O A harmonicales; ergo cum ſit QS = SP, & SP parallela F G, incidet punctum P (propter lemma 6. hujus p.) in rectam V O L; *i. e.* punctum P ſectioni & rectæ V O L commune eſt, ac proinde idem cum puncto T. Sed ob A Q R L (per præced.) harmonicè diviſam, erunt parallelæ rectæ A V, Q T, R O, harmonicales, & punctum L commune eſt; unde recta V O harmonicè dividitur à punctis V, T, O, L.

Coroll. 1. In hyperbolâ, vel ſectionibus oppoſitis, 49.
Si punctum V ita ſumatur ut ſit V O alterutri aſym- 50.
ptotôn D C parallela, manifeſtum eſt hanc uni tantùm ſectionum oppoſitarum idque in unico puncto T occurrere; unde in hoc caſu (occurſu L in infinitum abeunte) fit (per *Coroll*. 7. ad def. præced.) V T = T O. Patet etiam ut *coroll*. 4. prop. 1.

Coroll. 2. Et in Parabolâ, coincidentibus punctis V, 51.
A, hoc eſt, exiſtente V O diametro, fit V T = T O, ut prius oſtenſum.

Coroll. 3. Et in omnibus caſibus, puncto V in infinitum abeunte, hoc eſt, ductâ V T O L ipſi V A parallelâ, coincidentibus T, F, & G, L, fit T O = L O, quod & verum eſt ob diametrum A O.

Prop. IV. Probl. I.

A dato extra ſectionem, vel ſectiones oppoſitas, puncto 52. 53.
A, *rectas* A D, A G *ducere, quæ ſectionem vel ſectio-* 54. 55.
nes oppoſitas contingant. Oportet autem ut punctum A *in Hyperbolâ vel ſect. opp. non ſit centrum.*

Per A ducantur A I L, A *i l* ſectioni vel ſectionibus opp. vel utrique ſect. opp. &c. in I, L, *i*, *l* occurrentes, quod fieri poſſe manifeſtum eſt; Datiſque tribus harmonicæ diviſionis punctis in utrâque rectâ A, L, I; A, *l*, *i*, inveniatur in utrâque intra ſectionem quartum K, *k*; Connexa K *k* occurrat ſectioni, vel ſectionibus oppoſitis in D, G, erunt connexæ A D, A G contingentes quæſitæ.

Nam (per prop. 1.) conjungens tactus contingentium ex puncto A, tranſibit per K, *k*, proindeque non erit à D G diverſa.

Supple figuras horum caſuum.

Nota: Si ſit A I = A L, quod inter ſect. opp. quandoque fieri poteſt, invento *k*, ducenda eſt *k* D G parallela I A L; nam K infinitè diſtat. Si A ſit in alterutrâ aſymptotôn, erit D G infinita, *i. e.* aſymptoto parallela, unaque tantum duci poteſt contingens; hoc eſt, contingens altera erit ipſa aſymptotos.

Schol. Alias methodos (eaſque forte aliquando commodiores) ex prop. 1. hujus partis corollariis lector facile excogitabit.

Prop. V. Theor. IV.

56. 57. *In plano ſectionis conicæ, vel ſectionum oppoſitarum,*
58. 59. *ductâ quâvis rectâ* V A *quæ ſectioni, vel ſectionibus*
non occurrat, nec per centrum tranſeat; Inventâque
diametro A O M *rectarum in ſectione, vel ſectionibus*
ipſi A V *parallelarum, occurrente rectæ* A V *in* A;
Dico conjungentes tactus binarum quarumcunque contingentium V H, V I, *ex quovis puncto* V *rectæ* V A
ductarum, tranſire per unum idemque punctum O *in*
ſectione, vel in alterâ ſectionum oppoſitarum, medium
ſcilicet rectæ F G *conjungentis tactus contingentium*
A F, A G *à puncto* A *ductarum.*

Conjungens tactus F G (per def.) ordinata eſt ad diametrum A O M, proindeque parallela A V & biſecta in O; connexa V O & producta occurret ſectioni, vel ſectionibus opp. in T, L, vel (ſi ſit aſymptoto parallela) in unico puncto T. Si occurrat in binis punctis, eadem à punctis V, T, O, L harmonicè dividitur; aliàs à punctis V, T, O bifariam dividitur, ut patet ex prop. 3. & ejus *Coroll.* 1; at per prop. 1. & *Coroll.* 4. liquet conjungentem tactus H I contingentium V H, V I tranſire per O ; cùm manentibus cæteris punctis non ſit intra ſectionem aliud punctum diviſionis harmonicæ, vel diviſionis bifariam, ab O diverſum.

60. *Coroll.* 1. In hyperbolâ, Si punctum V in alteram aſymptotôn incidat, contingentium ex V alterâ V I in aſymptoton degenerante, recta per alterius V H contactum H eidem aſymptoto parallela per O tranſibit: Nam H I vice fungitur tactus conjungentis.

56. 57. &c. *Coroll.* 2. Si à puncto A extra ſectionem, vel ſectiones opp. ducantur duæ contingentes A F, A G, ſitque tactus conjungens F G ; ſimiliter ex alio puncto V ſint contingentes V H, V I, & tactus conjungens H I, occurrens F G in O ; Junctâ A V, erit punctum O concurſus omnium conjungentium tactus contingentium ex quovis puncto rectæ A V ductarum : & (per prop. 3. cum *Corollariis*) ductâ per O rectâ quâlibet occurrente utcunque rectæ A V, & ſectioni vel ſect. opp. in binis punctis, eadem harmonicè dividitur ; vel bifariam dividitur tantùm, ſi ſit alteri aſymptotôn hyperbolæ parallela, vel ſit parabolæ diameter.

Prop. VI. Theor. V.

56. 57. 58. 59. *Sumpto intra ſectionem conicam, vel unam ſectionum oppoſitarum quolibet puncto* O, *quod in Ellipſi non ſit centrum, inventâque diametro quæ per* O *tranſit*; *per* O *ad hanc ordinetur* F O G, *ſectioni in* F, G *occurrens, in cujus extremis ſectionem contingant* F A, G A, *diametro* A O *occurrentes in* A, *& per* A *agatur* A V *ipſi* F G *parallela*; *Dico rectas* H K, I K *contingentes in occurſibus rectæ cujuſvis* H O I *per punctum* O *ductæ, & ſectioni, vel ſect. opp. in* H, I *occurrentis, convenire in punctum aliquod rectæ* A V, *aut eſſe eidem parallelas.*

Si contingentes H K, I K concurrant in K, agatur K O, quæ producta occurrat ſectioni vel ſectionibus in L, T, (quod ſemper fiet niſi K O ſit alteri aſymptotôn hyperbolæ parallela) & rectæ A V in V; Erit (propter diametrum) O medium punctum rectæ F G, unde recta L O T K V (per prop. 1.) harmonicè dividitur à punctis L, O, T, K, & (per prop. 3.) à punctis L, O, T, V, ergo coincidunt puncta K, V. Si L O T K V ſit aſymptoto parallela biſariam dividitur à punctis O, T, V, & à punctis O, T, K, unde in hoc etiam caſu coincident K, V.

2. Si H O I diametro A O coincidat (in Ellipſi ſcilicet vel ſect. opp.) manifeſtum eſt fore H K, I K ipſi A V parallelas.

60. *Coroll.* Cum eadem ſit ratio rectæ per tactum aſymptoto parallelæ, atque tactus conjungentis; Si recta H O I ducta ſit alteri aſymptotôn parallela, Liquet tangentem in occurſu ejus cum ſectione H K, per ejuſdem aſymptoti occurſum cum rectâ A V tranſire.

Prop. VII. Theor. VI.

61. 62. 63. 64. *Supple caſus omiſſos.* *In omni Sectione Conicâ, vel ſectionibus oppoſitis, ductâ quâvis rectâ* F G *quæ ſectioni vel ſectionibus oppoſitis in binis punctis* F, G *occurrat* (*vel forte unico in hyperbolâ vel ſectionibus oppoſitis*) *nec per centrum tranſeat; Dico conjungentes tactus* H, I *binarum quarumcunque contingentium* B H, B I, *à quovis puncto* B *rectæ* F G *extra ſectionem vel ſectiones ſumpto, ad ſectionem vel ſectiones ductarum, tranſire per unum idemque punctum* A *extra ſectionem, vel ſectiones; punctum ſcilicet, in quod coeunt contingentes ſectionem, vel ſectiones, in punctis* F, G; *vel in quod tangens in horum punctorum altero convenit cum aſymptoto, quoties* F G *ducta eſt aſymptoto parallela.*
Supple hunc caſum. *Quòd ſi punctum* B *in alteram aſymptotôn incidat, recta per contactum unicæ contingentis à puncto* B *ductæ eidem aſymptoto parallela, per idem punctum* A *tranſibit.*

A puncto B ducatur B K L M ſectioni, vel ſectionibus oppoſitis pro libitu in K, M occurrens, & rectæ

HI ſi opus productæ in L, dicaturque rectarum F G, HI occurſus R, & jungantur K R, M R, quarum M R ſi opus producta occurat eidem, vel oppoſitæ ſectioni in N; erunt (propter BKLM harmonicè diviſam) rectæ BR, KR, LR, MR harmonicales: Ducta BN (ſi opus producta) occurrat LR (ſi opus productæ) in P, KR (ſi opus productæ) in Q, & ſectioni in O; hæc (per prop. 1. hujus p.) harmonicè dividitur in B, N, P, O, & propter harmonicales (per lemma 8.) in B, N, P, Q, coincidunt ergo puncta O, Q; ſive occurſus rectarum BN, KR in ipſam ſectionem incidit.

Ductis igitur KSN, MTO ſecantibus FG in S, T, hæ (per lemma 9.) convenient ſuper rectâ IH (ſi opus productâ) in punctum aliquod D; (nam ſi dicantur KN, HI, MO parallelæ, erit (per lemma 6.) KS = SN & OT = TM, unde FG tranſibit per centrum contra hypotheſin.) Sed ob harmonicales LRD, NRM, BFSRTG, KRO, rectæ DNSK, DOTM harmonicè dividuntur à punctis D,N,S,K; D,O,T,M; unde (per prop. 4.) junctæ DF, DG ſectionem vel ſectiones oppoſitas contingent; vel ſaltem continget altera, erit altera aſymptotos; *i.e.* non erunt ab AF, AG diverſæ. Tranſit ergo IH per A.

Si MR ſit aſymptoto parallela, ducendæ ſunt KN, BN huic parallelæ. Si GF, HI ſint parallelæ (quod in ſect. opp. fieri poteſt) ducendæ ſunt MR, KR his parallelæ. Si recta quævis ſit cuivis harmonicalium parallela, non harmonicè, ſed bifariam ſecabitur. Eademque quæ prius conſequentur, adhibitis (pro re natâ) lemmate 6, vel 10. & *coroll.* 4, vel 5. prop. 1.

Prop. VIII. Theor. VII.

Sumpto extra ſectionem conicam vel ſectiones oppoſitas 61. 62.
quolibet puncto A *quod non ſit centrum*; *Dico binas* 63. 64.
quaſcunque HB, IB *contingentes ſectionem vel ſectiones oppoſitas in duobus occurſibus rectæ cujuſvis* AHI *per punctum* A *ductæ, convenire in punctum aliquod*

aliquod B *rectæ* F G *conjungentis tactus rectarum* A F, A G *ſectionem vel ſectiones contingentium à puncto* A *ductarum; vel in punctum aliquod* B *rectæ quæ per contactum unicæ contingentis à puncto* A *ductæ alteri aſymptotôn parallela ducitur, quoties punctum* A *in eàdem aſymptoto ſumptum eſt.*

Quòd ſi recta A H I *ſit alteri aſymptotôn parallela, contingens in unico ejus occurſu cum ſectione per ejuſdem aſymptoti occurſum cum rectâ* B F G *tranſibit.*

Contingentes in H, I, coeant in B; ex A ductis contingentibus A F, A G, conjungens tactus F G, ſi opus producta (per prop. præced.) tranſibit per B. Et ſic (mutatis mutandis) in omnibus caſibus.

Nota. Si I H ſit diameter Ellipſeos vel ſect. opp. punctum B infinite diſtabit, hoc eſt, erunt I B, H B, G F B parallelæ.

Prop. IX. Theor. VIII.

65. 66. 67. 68. *Supple caſus omiſſos.*

Si ſectionem conicam vel ſectiones oppoſitas contingant duæ rectæ A F, A G *concurrentes in* A; *ſumptoque in tactus conjungente* F G *puncto* B, *ducantur totidem aliæ ſectionem vel ſectiones contingentes* B H, B I, *prioribus contingentibus in* K, L, *&* E, D *occurrentes: Dico has quatuor contingentes à mutuis occurſibus, propriiſque contactibus harmonicè dividi; ſcilicet à punctis* A, K, F, E; A, L, G, D; B, K, H, L; B, E, I, D; *vel bifariam dividi tantum quando occurſuum vel contactuum aliquis abit in infinitum.*

Aſymptoti contingentibus, hiſque parallelæ per tactus, tactus conjungentibus, (ut in prioribus) accenſentur.

Conjungens tactus H I (per prop. 7.) tranſit per A; Et propter B G, A I harmonicè diviſas à B, F, O, G; A, H, O, I punctis, aut ſaltem bifariam diviſas, Erunt A B, A F, A O, A G, item B A, B H, B O, B I harmonicales; Unde liquet propoſitum.

Coroll. Si tres tantùm ſint contingentes A F, A G, B H, quarum quælibet B H conjungenti tactus reliquarum occurrit in B, ipſis vero contingentibus in K, L; Dico hanc harmonicè dividi à punctis B, K, H, L, vel bifariam tantùm ſi &c. Ducta A H & producta occurrat ſectioni, vel ſectioni oppoſitæ in I; Contingentes in H & I coibunt (per prop. 8.) in aliquod punctum rectæ F G, quod erit punctum B, in quod B H, F G prius coibant; Redit itaque caſus in illum hujus prop. 9. Et ſic in reliquis contingentibus, & in omnibus caſibus. 65. 66. 67. 68. *Supple caſus omiſſos.*

Schol. In parallelis contingentibus B H, A G in Ellipſi & opp. ſect. tranſit caſus hujus corollarii in illum *coroll.* Prop. 27. partis 1. 67.

Prop. X. Theor. IX.

In Ellipſi, & Sectionibus oppoſitis, ſi à terminis cujuſvis diametri A B *ducantur contingentes* A K, B I, *cuivis aliæ contingenti* T D *occurrentes in* K, I; *Dico* $AK \times BI$ *æquale eſſe quartæ parti figuræ diametri* A B, *i. e.* (*ſi ſit centrum* C *& parameter ſive latus rectum* l r) $CA \times \tfrac{1}{2} lr = AK \times BI$. 69. 70.

Contingens T D occurrat diametro in D, à contactu T ordinetur ad diametrum recta T O, & à centro C agatur ad contingentem T D recta C R rectis A K, B I parallela: Erit per *Coroll.* 2. prop. 1.

1. $CD : CA : CO$ ∺ id eſt,

$CD : CO :: CAq : COq.$

Et dividendo

$$2.\left\{\begin{matrix} CO - CD \\ \text{vel } CD - CO \\ DO \end{matrix}\right\} : CD :: \left\{\begin{matrix} COq - CAq \\ \text{vel } CAq - COq \\ BO \times OA \end{matrix}\right\} : CAq;$$

Et per *Coroll.* 5. Prop. 24. Part. 1.

$$BO \times OA : OTq :: BA : lr :: \left\{\begin{matrix} \tfrac{1}{2} BA \\ CA \end{matrix}\right\} : \left\{ \tfrac{1}{2} lr \right.$$

$:: CAq : \left\{\begin{matrix} CA \times \frac{1}{2} lr \\ \frac{1}{4} \text{ fig. diam. } AB \end{matrix}\right\}$ vel altern.

3. $BO \times OA : CAq :: OTq : \frac{1}{4}$ figuræ diam. AB:

Ergo (per Proport. 2, & 3.)

4. $DO : CD :: OTq : \frac{1}{4}$ figuræ diam. AB.

Per *Coroll* 2. prop. 1.

$DA : DO :: DC : DB$, ideoque ob ſim. triang.

5. $AK : OT :: CR : BI$, i. e. $OT \times CR = AK \times BI$.

Rurſus ob ſim. triang.

6. $OT : CR :: DO : DC :: OTq : \left\{\begin{matrix} OT \times CR \\ AK \times BI \end{matrix}\right\}$

Ergo per proport. 4. & 6.

$OTq : \frac{1}{4}$ fig.diam. $AB :: OTq : AK \times BI$: unde $\frac{1}{4}$ fig. diam. $AB = AK \times BI$.

Si in Ellipſi contingens KTI ſit diametro AB parallela; Sit CN ſemidiameter huic conjugata, & coincident puncta T, R, N, fietque $AK = CN = BI$, adeoque $AK \times BI = CNq =$ (per *coroll.* 4. prop.24. p. 1.) $\frac{1}{4}$ fig. diam. AB.

69. 70. *Coroll.* 1. Poſitâ CN ſemidiametro ipſi AB conjugatâ, erit ubique $AK \times BI = \frac{1}{4}$ fig. diam. $AB =$ (per jam dictum *coroll.*) CNq.

69. 70. *Coroll.* 2. Et ſi ductâ diametro TCL, contingens in L occurrat IB productæ in M, erit $MB \times BI = \frac{1}{4}$ fig. diam. $AB = CNq$. Nam (ob parallelas AK, MBI, & KTI, LM; & æquales TC, CL & BC, CA,) æquiangulæ & æquales erunt figuræ CAKT, CBML; Unde $MB = AK$, & $MB \times BI = AK \times BI = CNq$.

Prop. XI. Theor. X.

71. 72. *Sint Ellipſeos, vel Hyperbolæ, aut ſectionum oppoſitarum, duæ quævis ſemidiametri conjugatæ* AC, CN, *quævis aliæ pariter conjugatæ* TC, CX, *ductiſque* AV, NV; TS, XS, *compleantur parallelogramma* ACNV, TCXS; *Dico hæc eſſe inter ſe æqualia.*

Producta A C ſectioni vel ſect. opp. occurrat in B, ductaque B I parallela C N occurrat T S in I, ipſa vero T S occurrat A V in K & A B (ſi opus productæ) in D; à centro C ducatur C Q occurrens T S in Q, ut ſit ang. D A K vel D C N = D Q C, agaturque huic parallela X Y occurrens T S in Y, & à T ordinetur ad diametrum A B recta T O: Erunt rectæ T S, A K, B I contingentes, & triangula D A K, D O T, D Q C, D B I, ſimilia.

Per ſim. triang. & *Coroll.* 2. prop. 1.

I D : T D :: B D : O D :: C D : A D, ideoque div. vel comp.

$$ID : \left\{\begin{matrix} ID \pm TD \\ IT \end{matrix}\right\} :: CD : \left\{\begin{matrix} CD \pm AD \\ CA \end{matrix}\right\} \; \&$$

altern.

1. I T : C A :: I D : C D :: (ob ſim. triang.) B I : C Q.

Rurſus per ſim. triang. & *coroll.* 2. prop. 1.

K D : T D :: A D : O D :: C D : B D, ideoque

$$KD : \left\{\begin{matrix} TD - KD \\ TK \end{matrix}\right\} :: CD : \left\{\begin{matrix} BD - CD \\ CB \end{matrix}\right\} \; \&$$

altern.

2. T K : C B :: K D : C D :: (ob ſim. tri.) A K : C Q.

Per proport. 1, & 2. ductis &c. & per *coroll.* 1. & 2. prop. præced.

$$\left.\begin{matrix} IT \times TK \\ CXq \end{matrix}\right\} : \left\{\begin{matrix} CA \times CB \\ CAq \end{matrix}\right\} :: \left\{\begin{matrix} BI \times AK \\ CNq \end{matrix}\right\} : CQq$$

Unde C X : C A :: C N : C Q; & C X × C Q = C A × C N. Sunt autem parallelogramma A C N V, C X Y Q (ob ang. D A K = D Q C) æquiangula, proindeque ad invicem ut rectangula ex lateribus, viz. C A × C N, C X × C Q, hoc eſt æqualia; ſed parallelogrammum T C X S (propter communem baſim & altitudinem æqualem) eſt parallelogrammo C X Y Q, adeoque ipſi A C N V, æquale

Coroll. Hinc parallelogramma circa ipſas diametros ſunt æqualia, utpote horum quadrupla.

Scholium. In ſectionibus opp. vel hyperbolâ, hæc propoſitio nullo negotio patet ex *coroll.* 2. prop. 28. p. 1.

71. Nam ſi tangentes in A, T occurrant aſymptotis in V, F ; S, E, erit (per def.) ſemidiameter $CN = AV = AF$, & $CX = TS = TE$, & $CN \parallel AV$, $CX \parallel ST$; ideoque parallelogr. $ACNV =$ triang. $CVF =$ triang. $CSE =$ parallelogr. TCXS. Sed cùm præcedens demonſtratio (utpote generaliori proprietati innixa) ad haſce ſectiones cum Ellipſi ſe juxtà extenderet, congruum erat & ipſas pariter eâdem complecti.

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PARS, II. TAB. X.

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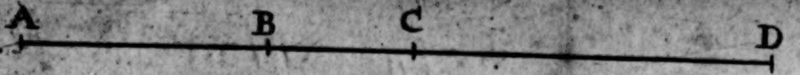

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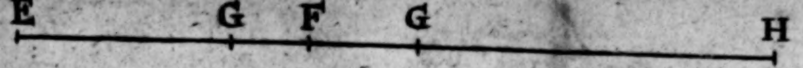

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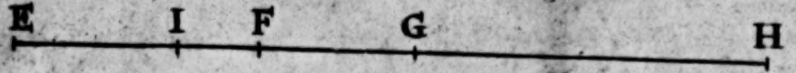

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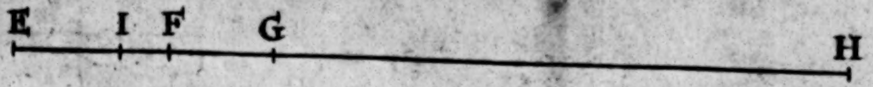

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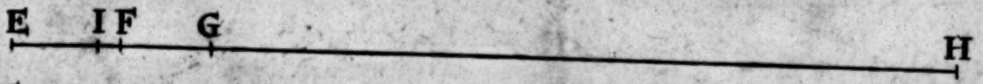

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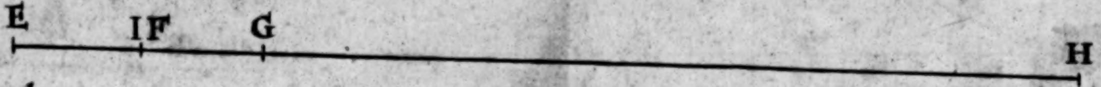

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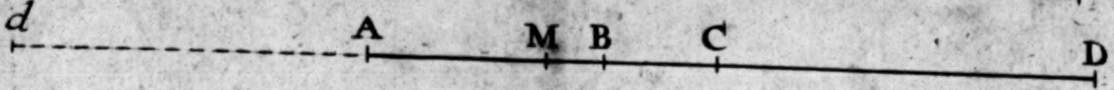

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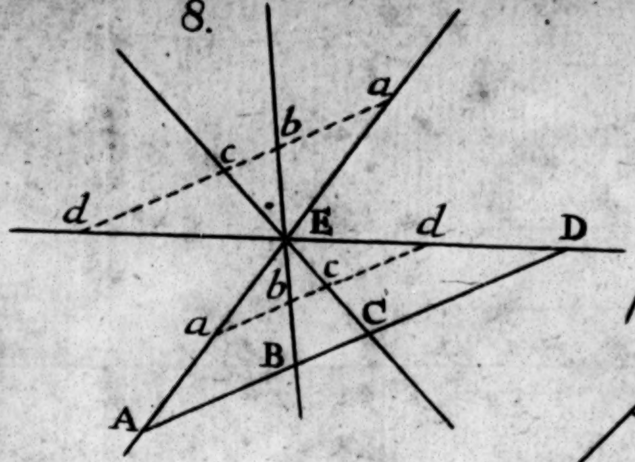

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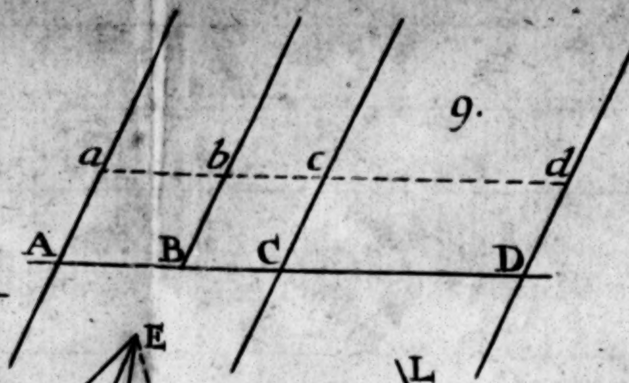

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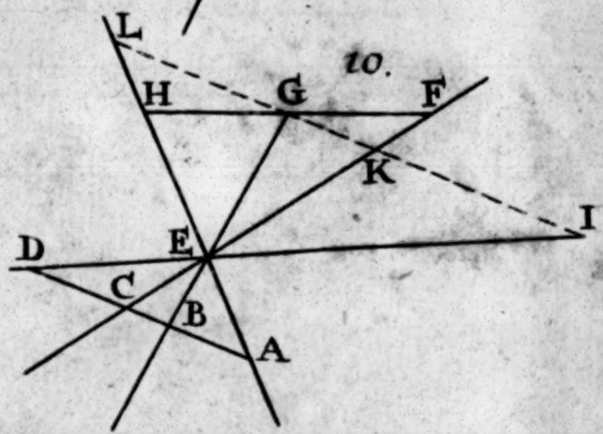

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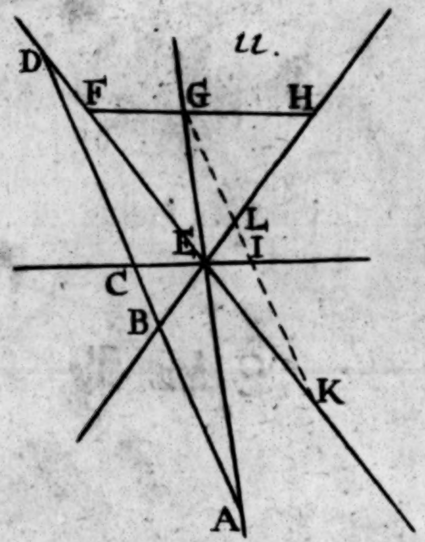

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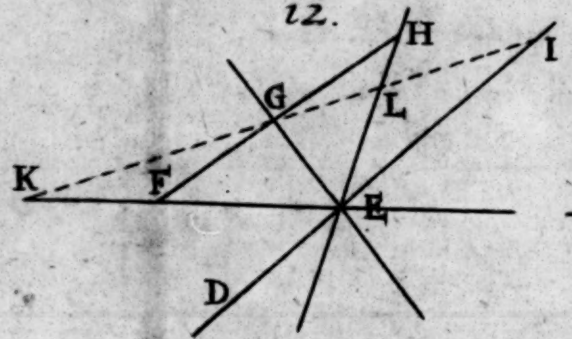

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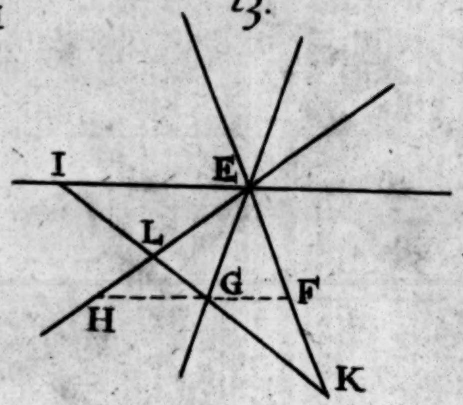

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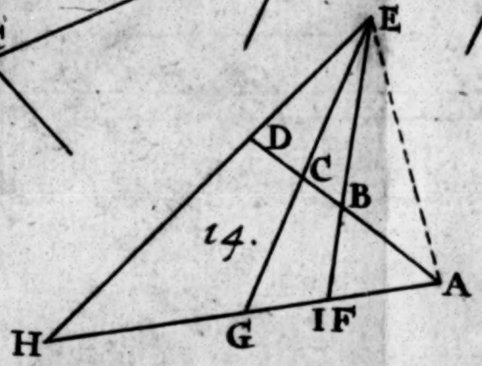

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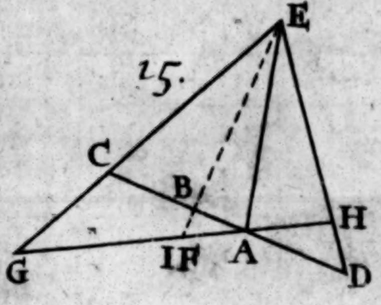

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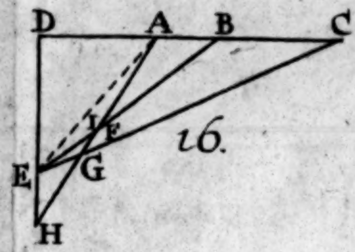

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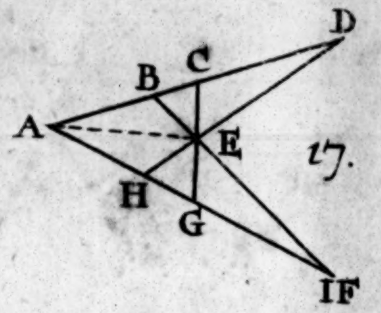

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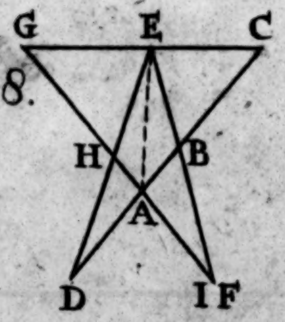

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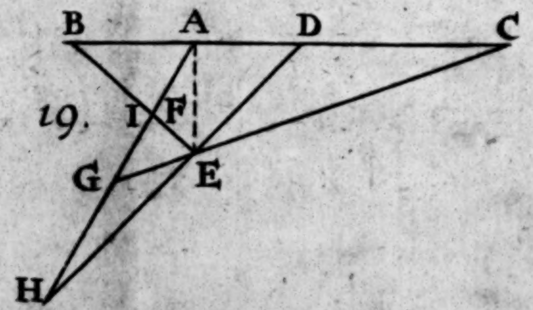

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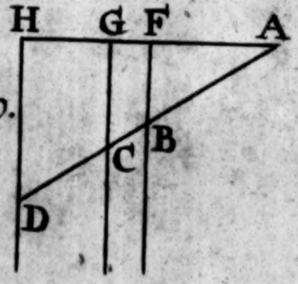

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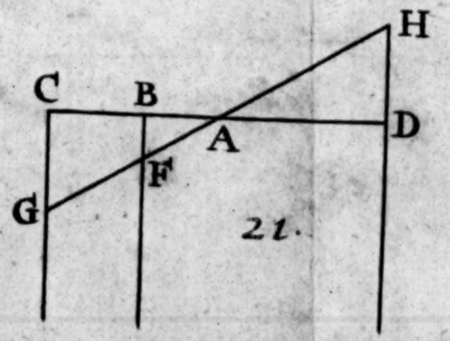

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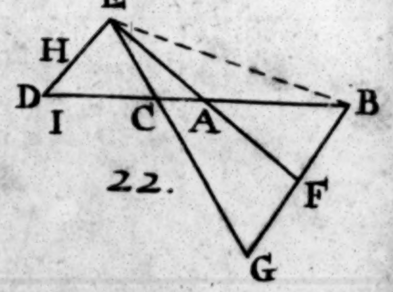

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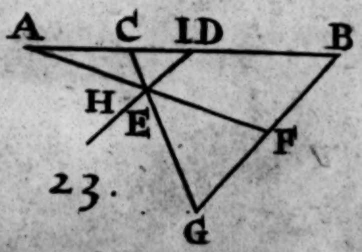

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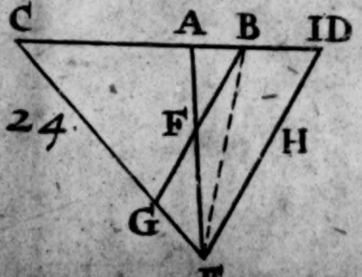

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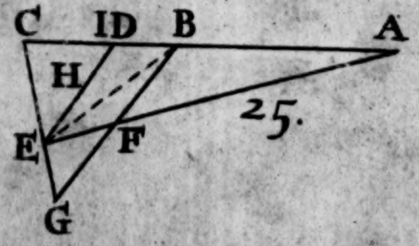

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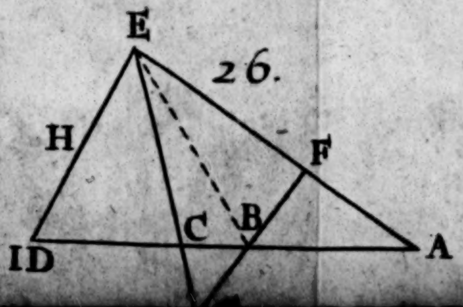

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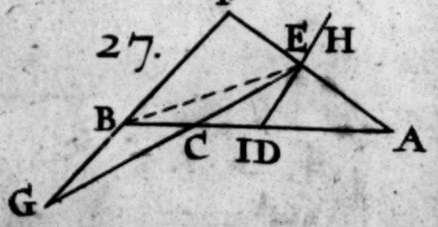

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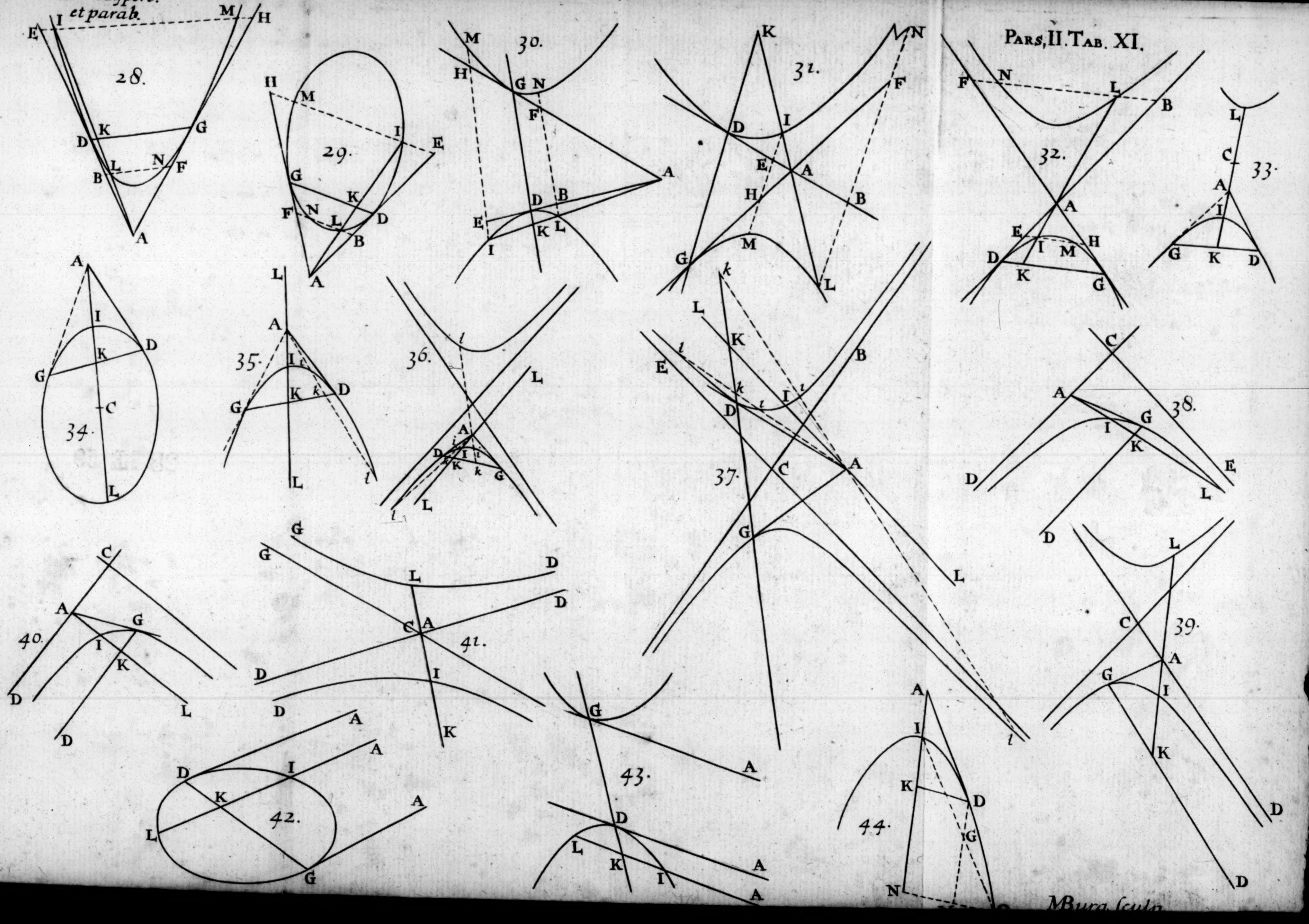

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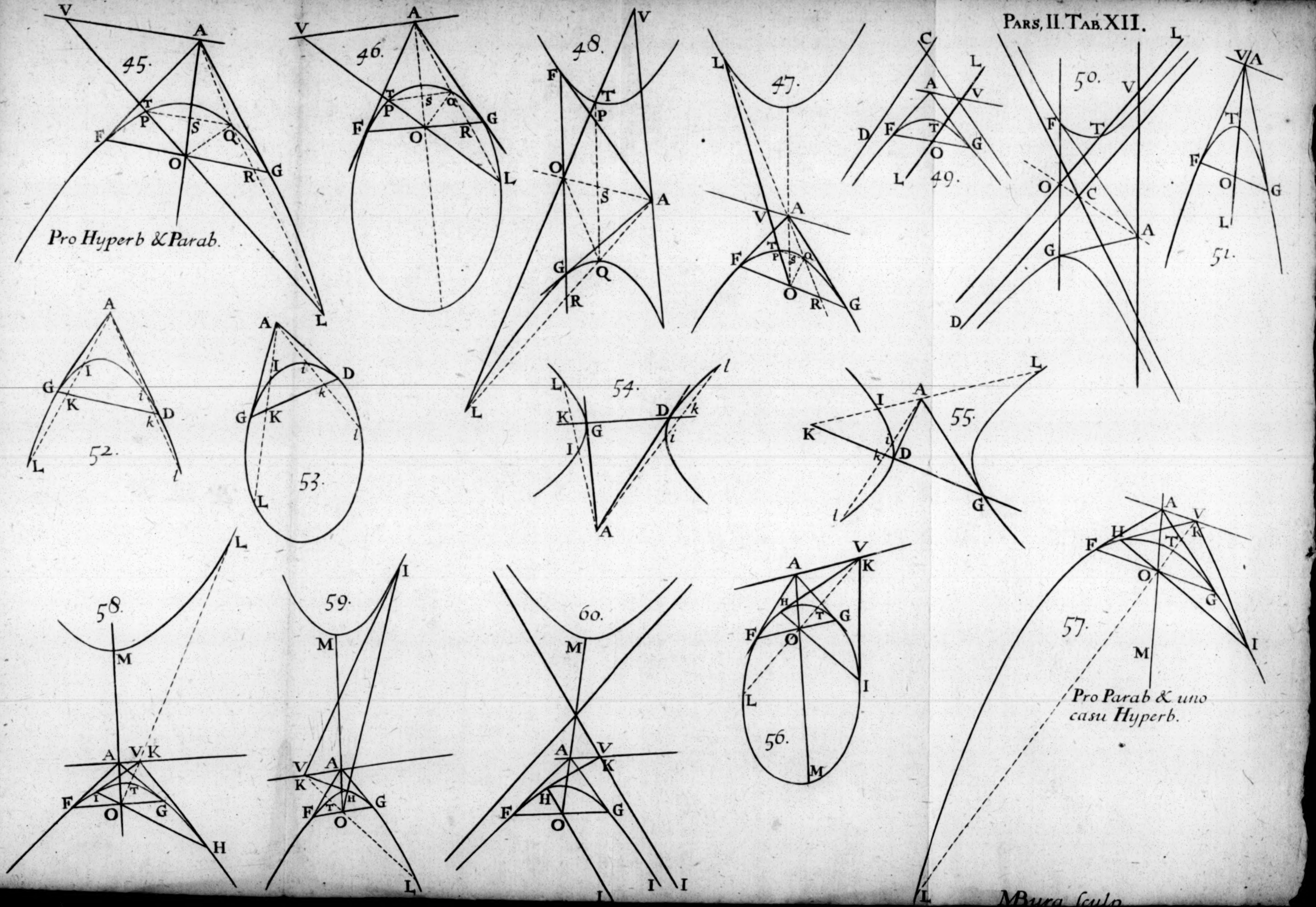

Pro Hyperb & Parab.

Pro Parab & uno casu Hyperb.

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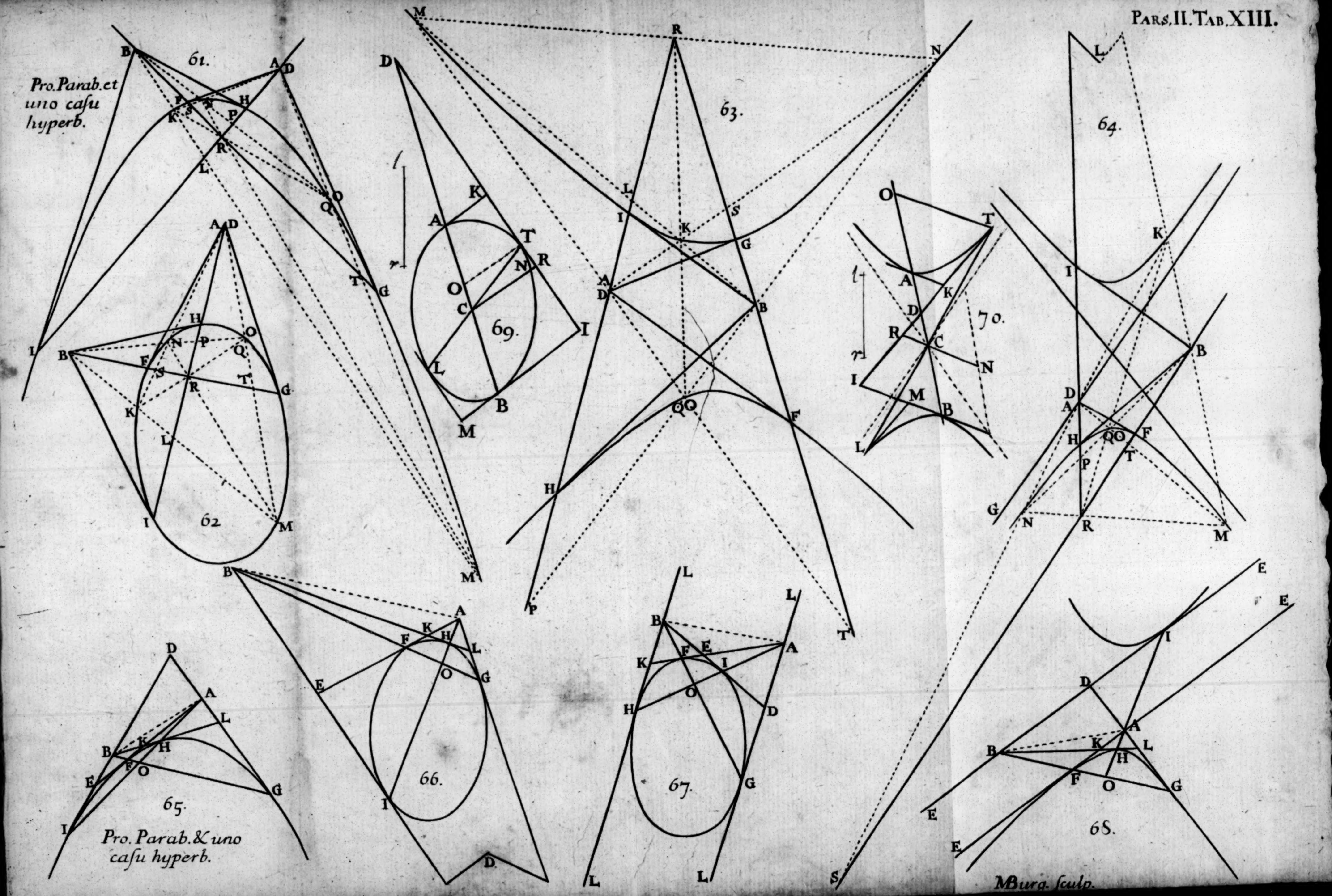
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PARS III.

Propoſitio I. Theorema I.

Circulus ſectioni conicæ, quæ ipſa non ſit circulus, vel 1.
ſectionibus oppoſitis, ad quatuor plurima puncta A,
B, C, D *occurrit*; *Ad tria* A, B, C, *quorum unum* A, 2.
Ad duo tantùm A,B, *quorum utrumque eſt contactus.* 3.

NAM ſi ſectio [vel ſect. opp.] circulo congrueret, eſſet circulus contra hypotheſin; Patet ergò per *Coroll.* 5. prop. 35. p. 1.

Def. In omni ſectione conicâ, vel ſectionibus op- 4. 5.
poſitis, diametri quibus ordinatæ ſuæ ad rectos angu- 6.
los inſiſtunt, ut A B, C D, *Axes* appellantur.

Prop. II. Probl. I.

In Hyperbolâ vel ſect. opp. & in Ellipſi quæ non ſit cir- 7. 8.
culus, Axes invenire; Et duos tantum eſſe axes oſtendere.

Inventâ quâvis diametro A B, quæ in hyperbolâ vel ſect. opp. ſit determinata, Diametro A B (*i. e.* centro C, intervallo C A, vel C B) deſcribatur circulus A M B G; Hujus peripheria vel ſectionem aut ſect. opp. in A, & B, continget, (quo in caſu per prop. præced. ei non ampliùs occurret;) vel iterum occurret ſectioni, vel ſectionibus opp. in binis punctis M, G.

Si contingat in A & B, erunt ipſa A B atque huic conjugata, Axes quæſiti. Nam contingentes circulum H A I, K B R in hoc caſu contingent etiam ſectionem vel ſect. opp. eſtque propter circulum angulus B A I,
vel

vel B A H rectus, & diametri A B conjugata erit parallela A I, ſuntque diametri conjugatæ ad ſe mutuò ordinatæ, proindeque hæ diametri ſibi mutuò ad angulos rectos inſiſtent.

Si A, B non ſint contactus; Ex punctorum A, B utrovis A ad punctorum M, G utrumvis M agatur A M; Biſectâ A M in N, & per centrum C ductâ C N, erunt diameter C N atque huic conjugata Axes quæſiti.

Prior caſus jam ſatis liquet. In ſecundo probandum primò eſt circuli peripheriam ſectioni, vel ſectionibus oppoſitis in M, G occurrere, deinde angulum C N A eſſe rectum.

Sectionem vel ſect. opp. in A, B tangant D A E, F B T, hæ (per *Coroll.* 8. prop. 20. part. 1.) erunt parallelæ; cúmque occurſus A, B non ſint tactus, erunt hæ à contingentibus circulum K B R, H A I, diverſæ; Unde contingens ſectionem B F tranſibit intra circulum, & contingens circulum H A tranſibit intra ſectionem, hoc eſt, ſemicirculus B M A ad partes B eſt extra ſectionem, vel utramque ſectionum opp. ad partes vero A eſt intra ſectionem, vel unam ſectionum opp. unde neceſſariò occurret ei alicubi in M. Pari ratione alter ſemicirculus occurrit ſectioni ex alterâ parte, vel ſectionum opp. alteri in G.

Connexâ C M, erunt (propter circulum) C A, C M æquales, eſtque (ex conſtruct.) M N = N A, unde Ang. C N A eſt rectus, & (per *cor.* 2. prop. 22. part 1.) L C N O eſt rectæ M N A diameter, Eſtque huic conjugata rectæ M N A parallela, ſuntque conjugatæ diametri ad ſe mutuò ordinatæ, proindeque in noſtro caſu ſibi mutuò ad angulos rectos inſiſtunt.

9. 10. 2. Si jam, præter diametros O C L atque huic conjugatam Y C X hâc methodo inventas, alia quævis T C V dicatur Axis; à quovis punctò ſectionis vel utriuſvis ſectionum opp. A, ad T C V, & ad unum ex axibus primò inventis O C L, ſint ordinatæ A S R, A N M, & ducantur diametri A C B, R C Q, M C P; Ob A S = S R, & ang. A S C (ex hypoth.) rectum, erit A C = C R;

Pari modo erit $AC = CM = CP = CB =$ (prius) $CR = CQ$; Unde (ob has rectas æquales) circulus centro C tranſire poteſt per ſex puncta ſectionis, vel ſect. opp. P, Q, B, A, R, M, contra prop. præced. Unde diameter T V non erit axis, neque huic conjugata.

Coroll. 1. In Ellipſi, conjugati axes tranſverſi L C O, 11.
X C Y ſunt inæquales. Aliàs, connexâ L Y & biſectâ in D, ductâque diametro B C D E, eſſèt (ob $LC = CY$) angulus L D C rectus, cujus contrarium jam oſtendimus.

Coroll. 2. In Hyperbolâ Axis tranſverſus O C L eſt 12.
minima omnium determinatarum diametrorum, & huic propiores ſunt remotioribus minores. Nam ſi T V dicatur minor quam L O, ſemicirculus diametro L O ſecabit T V in Z alicubi intra unam ſectionum opp. & eidem ſectioni occurret denuò in A; unde diameter B C P quæ biſecat junctam O A in P, (propter $CA = CO$, & $OP = PA$) erit axis, contra jam probata. Pari prorſus modo oſtendetur T C V eſſè remotiore quâvis diametro minorem.

Coroll. 3. In Ellipſi axis major eſt omnium diametro- 13.
rum maxima, minor vero minima: & minori propiores ſunt remotioribus minores; majori, majores. Nam ſi T V dicatur minor quam Y X; Semicirculus diametro Y X ſecabit T V extra ſectionem in Z, ſecabit vero axem majorem (propter $CO \sqsubset CX$) alicubi intra ſectionem in F; occurret itaque ſectioni alicubi in A; unde oſtendetur ut in præcedenti *Corollario* diametrum B C P quæ connexam X A in P biſecat, eſſè axem, ab ipſis O L, Y X diverſum, contra jam oſtenſa. Eodem modo oſtendetur majorem axem O L eſſè quâvis aliâ diametro T V majorem, & majori propiores remotioribus majores eſſe, minori minores.

Coroll. 4. In hyperbolâ vel opp. ſect. & Ellipſi, Dia- 14. 15.
metri M C G, A C B ductæ ab extremis ordinatæ cujuſ- 16.
vis M A ad utrumvis axem L O ſunt æquales; Idem intellige de contingentibus M E, A E in punctis M, A, ſuper

per axi (per *Coroll.* 17. prop. 20. p. 1.) in puncto aliquo E coeuntibus. Patet ob M N = N A, ob communes E N, N C, & angulos ad N rectos.

14. 15. 16. *Coroll.* 5. In iiſdem ſectionibus, Recta E C bifariam dividens angulum vel à binis æqualibus contingentibus, vel binis æqualibus diametris factum, erit Axis. Nam biſecabit M A, eidemque ad angulos rectos inſiſtet. In hyperbolâ vel opp. ſect. rectæ, quæ bifariam dividunt utrumque angulum aſymptotôn, ſunt Axes conjugati.

14. *Coroll.* 6. Et quarum diametrorum angulum utervis axis bifariam dividit, eædem ſunt æquales. Nam in Ellipſi *ex. gr.* Si dicas C Q (⊐ vel ⊏ C M) = C A: Junctâ A Q ſecante axem in P, & Sectionem in R; ob æquales angulos Q C P, A C P, & C A = C Q, erit A P = P Q, & angulus ad P rectus; unde A P R eſt ad axem ordinata, hoc eſt A P = P R = (priùs) P Q, quod abſurdum eſt. Et eodem modo (ductâ hujuſmodi A P Q R) in hyperbolâ vel ſect. opp. hoc *Corollarium* demonſtrabitur.

14. 15. 16. *Coroll.* 7. Quæ diametri M C G, A C B æqualiter ab utrovis axe diſtant, æquales faciunt angulos ad contingentes in earum verticibus C M E, C A E. Nam triangula C M E, C A E ſimilia & æqualia ſunt.

Coroll. 8. Ex *Coroll.* 2, & 3. Liquet Circulum ſuper axi determinato Hyperbolæ vel ſectionum oppoſitarum, tanquam diametro, deſcriptum, totum eſſe extra utramque ſectionem: Super Ellipſeos axi majore, ſectionem intra ſe continere: Super Ellipſeos axe minori, à ſectione contineri.

Prop. III. Probl. II.

17. *Invenire Axem Parabolæ: Et unicum ejus eſſe axem oſtendere.*

Inventâ quâlibet diametro C D, ſumptoque in eâdem ubivis puncto D, erigatur ad eandem perpendicularis E D F occurrens ſectioni in E, F; Biſectâ E F in B, ductâque B A parallelâ C D, Erit BA axis quæſitus. Nam (per *Coroll.* 16. Prop. 20. p. 1.) B A eſt rectæ E F diameter, & (ob rectas parallelas) erit angulus A B F rectus.

Si punctum D biſecet E F, ipſa diameter C D primò inventa eſt axis.

2. Si dicas alium eſſe axem C D ab A B diverſum: Ordinatâ ad hunc F D E ſecante axem primò inventum in B; Ob paralleliſmum diametrorum erit E F ad utrumque axem C D, A B perpendicularis, hoc eſt, ad utrumque ordinata, ideoque tam in B quam in D biſecta, quod abſurdum eſt.

Coroll. 1. Contingentes M E, A E in extremis rectæ cujuſvis M A ad axem ordinatæ ſunt æquales. 18.

Coroll. 2. Recta bifariam dividens angulum ab æqualibus contingentibus factum eſt axis. Patet utrumque ſicut in hyperbolâ vel ſect. opp.

Coroll. 3. Diametri M G, A B æqualiter ab axe diſtantes æquales faciunt angulos ad contingentes in earum verticibus, viz. G M E, B A E. Patet ob ſimilia & æqualia triangula E M N, E A N, & angulos G M N, B A N rectos.

Coroll. 4. Angulus E M G, quem quævis diameter facit cum contingente in ejus vertice verſus axem, obtuſus eſt, utpote recto G M N major: Et quo diameter M G ab axe remotior eſt, eo hujuſmodi angulus major eſt; Nam contingens *e m* in vertice propioris diametri *m g* producta occurret E M & E O extra ſectionem in P, *e*; ductâque *m n* parallelâ M N, erit ang. $PeN =$ ang. $PEN + EPe$, unde $PeN \sqsubset PEN$, & $emn \sqsupset EMN$, additiſque utrinque rectis, $emg \sqsupset EMG$.

Prop. IV. Theor. II.

In Ellipſi diametri I C H, K C G *rectarum* A E, A D *utriuſque axis extrema conjungentium ſunt conjugatæ, & æquales; & præter ipſas non ſunt aliæ diametri conjugatæ æquales.* 19.

19. Junctis EB, DB, ob CD = CE & CA = CB & angulos ad centrum rectos, erit AEBD figura parallelogramma cujus latera æqualia; unde diametri, IH, KG biſecant etiam EB, DB, ac propterea erunt rectis EA, AD reſpectivè parallelæ; Hoc eſt, erunt diametri IH, KG conjugatæ: Erit porro AFCN (ob AF ∥ CN & AF = AN, utpote æqualium AD, AE dimidia) figura parallelogramma cujus latera æqualia; Unde axis ACB bifariam dividet ang. FCN; ſunt ergo (per *Coroll.* 6. Prop. 2) diametri KG, IH æquales.

20. 2. Stantibus quæ in priore figurâ, Sit AB axis major, ED minor, & OP, QR duæ quævis diametri conjugatæ, ab ipſis HI, GK & ab axibus diverſæ; Harum una QR dividat *ex. gr.* angulum DCK, & in R contingat TRL: Hæc (per *Coroll*, 6. Prop. 23. p. 1.) occurret diametris IH, GK productis in L, T, proindeque huic parallela OP cadet extra angulum HCK; pari vero ratione eadem OP cadet extra angulum DCB, hoc eſt intra angulum ACD; unde dividet neceſſariò angulum ACH; Eſt itaque (per *Cor.* 3. prop. 2.) OP $\sqsubset$ HI, & HI = GK $\sqsubset$ QR, hoc eſt, OP $\sqsubset$ QR; ideoque OP, QR inæquales.

20. *Coroll.* 1. In Ellipſi majoris diametri tranſverſæ minor erit ſecunda huic conjugata, minoris major. Nam referant jam GK, HI duas quaſcunque diametros conjugatas, ſive æquales, ſive inæquales, OP, QR alias quaſcunque pariter conjugatas; ſintque axes AB major, ED minor, ut priùs: Et ſi QR dividat *ex.gr.* an-angulum DCK, oſtendetur (ut priùs) OP dividere angulum ACH; unde (per *coroll.* 3 prop. 2.) erit OP $\sqsubset$ HI & GK $\sqsubset$ QR. Contrarium accidit in hyperbolâ, vel ſect. opp. ut ſuo loco oſtendemus.

Coroll. 2. Hinc ſi duæ contingentes Ellipſim QM, GM concurrant in M, contingens QM in vertice minoris diametri QR major erit quàm contingens GM in vertice majoris diametri GK. Nam QM, GM, ſunt diametrorum QR, GK conjugatis OP, HI reſpectivè parallelæ; Eſtque (per prop.17, vel 18. p. 1.)

$QMq : MGq :: OC \times CP = OCq : HC \times CI = HCq$; Eſtque (per *coroll.* præced.) $OP \sqsubset HI$, hoc eſt $OC \sqsubset HC$; unde $QM \sqsubset GM$.

Schol. Cum in Hyperbolâ (non æquilaterâ) ſemidiameter tranſverſa C B ſemidiametro ſecundæ C D, hoc eſt ſemicontingenti B L, ſit ubique inæqualis; Vertice vero B in infinitum abeunte & coincidentibus punctis L, C, rectæ C B, B L aſymptoto & ſibi invicem coincidant, hoc eſt, tum demum fiant æquales: Reſpondebunt aliquatenus Hyperbolæ aſymptoti Ellipſeos diametris conjugatis æqualibus; ſed cum hoc diſcrimine, quod Ellipſeos diametri ſint altera alterius, utravis vero aſymptotos ſit quaſi ſui ipſius conjugata. *Vide fig. 90. partis primæ.*

In Hyperbolâ æquilaterâ diametri conjugatæ ſunt ubique æquales; uti etiam in Ellipſi, ſi Ellipſis ſit circulus; Nam quod circulus eſt inter ellipſes, id hyperbola æquilatera (quoad proprietates ſaltem nonnullas) inter hyperbolas.

Prop. V. Probl. III.

In Hyperbolâ, & inter ſectiones opp. & in Ellipſi, invenire diametros quæ cum ſuis ordinatis angulum faciunt dato cuivis angulo non recto Q, *vel ejus complemento ad duos rectos, æqualem; qui tamen in Ellipſi non ſit major eo quem faciunt rectæ* B D, A D *ab utroque extremo axis majoris ad utrumvis extremum minoris ductæ, nec minor eo quem faciunt rectæ ab utroque extremo axis minoris ad utrumvis extremum majoris ductæ.* 21. 22.

Super utrovis axe Ellipſeos, vel determinato hyperbolæ, vel ſect. opp. A B, tanquam chordâ, ſtatuatur arcus circuli A E B capiens angulum dato angulo Q vel ipſius complemento æqualem, qui (ſi opus) poſt A B continuetur; occurret hic Sectioni, vel utrique Sectionum oppoſitarum (præter A, B) in duobus punctis F, R; ſumpto horum utrovis F, connectantur A F,

B F,

B F, quibus biſariam diviſis in L, O, ductiſque diametris H L C M, N O C P; Hæ duæ diametri, totidemque aliæ eodem modo ope puncti R reperiendæ, propoſito ſatisfacient.

21. In Ellipſi ſit (exempli gratiâ) A B axis major, Sectionem contingat I A, in A; circulum contingat recta G A; Erit (propter axem) ang. I A B rectus; cumque arcus A E B (ex hypoth.) non ſit anguli recti capax, contingens G A cum rectâ B A faciet ex unâ parte angulum acutum, tranſibitque ideo ex eâdem parte intra ſectionem, hoc eſt, arcus ipſe tranſibit intra ſectionem: Rurſum arcus ſuper chordâ A B capiens angulum æqualem A D B, tranſibit per D; ergo arcus qui (ex hypoth) capit angulum minorem quàm A D B ſecabit axem minorem extra Sectionem in E; Eſt ergo idem arcus ad partes A intra Sectionem, ad partes vero E extra eandem; ſecabit ergo illam alicubi in F inter A, E; Pari ratione ſecabit eandem in R inter E, B. Idemque erit (mutatis mutandis) in axe minori.

22. In Hyperbolâ vel ſect. opp. contingat I A unam è ſectionibus oppoſitis, & A G circulum: Ob angulum B A I rectum & B A G obliquum, non coincident I A, A G, ſed angulum efficient; unde recta quævis angulum I A G dividens, tranſibit tam intra circulum quàm ſectionem, hoc eſt, circuli circumferentia ad puncta A, B (nam utriuſque eadem eſt ratio) tranſit intra ſectiones, quæ (cum in ſe revolvatur) utrique neceſſariò occurret denuò in F, R.

21. 22. Jam in utrâque figurâ, propter F L = L A & B C = C A, erit C L parallela B F & A F parallela C O; unde ang. C L A = C O B = B F A = dato Q, vel ejus complemento. Idemque erit de diametris ope puncti R repertis.

Coroll. 1. In hyperbolâ vel opp. ſect. manifeſtum eſt angulum Q ſumi poſſe dato cuivis obtuſo vel acuto æqualem; Angulumque quem determinata aliqua diameter facit cum ſuis ordinatis, vel cum contingente in ejus vertice, eo obliquiorem eſſe quo magis eadem diameter

meter ab axe determinato diſtat; & pariter in diametro huic conjugatâ. In Ellipſi hujuſmodi angulus ab utrovis axe ad utramvis diametrorum conjugatarum æqualium fit ſemper obliquior, atque inde ad alterum axem continuò ad rectum vergit. Nam in hyperbolâ vel ſect. opp. angulus B F A = C L A, quo punctum F ab A remotius eſt, eo continuò minor eſt, donec abeunte puncto F in infinitum, dato quovis angulo minor fiat. In Ellipſi quò punctum F ab A remotius eſt, eo angulus B F A major eſt; donec, coincidente puncto F ipſi D, fit C L una diametrorum æqualium; atque inde ad B fit idem angulus continuò minor, donec, coincidentibus punctis F, B, recta A F, hoc eſt jam A B, fit ad axem minorem ordinata. Quâ in re, ulteriorem æqualium Ellipſeos diametrorum cum Hyperbolæ vel ſect. opp. aſymptotis analogiam obſervare licet.

Coroll. 2. In Ellipſi, liquet angulum C L A vel C O B à ſemiaxe majore ſubtenſum eſſe obtuſum; Nam huic æqualis B F A inſiſtit arcui B A ſemicirculo majori: In hyperbolâ liquet angulum C L A vel C O B eſſe acutum, nam huic æqualis B F A inſiſtit arcui A E B ſemicirculo minori. Idem intellige de angulis quos ad eaſdem partes hæ diametri faciunt cum contingentibus in earum verticibus; ſunt enim hæ contingentes ordinatis parallelæ.

Prop. VI. Probl. IV.

Idem in Parabola præſtare.

23. Inventâ quâlibet diametro K B L, per punctum ejus intra ſectionem B ad hanc inclinentur duæ rectæ A B I C, D B G E in angulis K B E, L B C dato Q æqualibus, occurrentes ſectioni in A, C; D, E; Biſecentur A C, D E in I, G; per I, G agantur diametri H I, F G. Propter diametrorum paralleliſmum erunt anguli H I B, F G E æquales dato Q; unde propoſito ſatisfit.

Coroll. Manifeſtum eſt (per *Coroll.* 3, & 4. prop. 3.) diametros F G, H I ab axe M N æqualiter utrinque diſtare; & propterea quod hujuſmodi rectæ A B C, D B E in quocunque angulo ad diametrum K B inclinatæ ſemper utrinque ſectioni occurrant, angulum Q, ſicut in hyperbolâ, vel ſect. opp. cujuſvis magnitudinis ſumi poſſe.

Prop. VII. Theor. III.

24. *In Hyperbolâ ſecunda diameter axi conjugata, vel quod idem eſt, contingens in ejus vertice* E B D *aſymptotis terminata, minor eſt quàm contingens in vertice cujuſlibet alterius diametri ut* H F G; *& quo quælibet diameter eſt axi propior, eo ejuſmodi contingens minor eſt, major quo remotior.*

Per tactum F & utrumvis rectæ G H extremum *ex. gr.* H, agantur F N L, H I K parallelæ E B D, ſecantes axim in N, I, & aſymptoton C G in L, K, & connectatur L I. Ob G F = F H, erit H K = 2 F L; & ob E B = B D, erit I H = I K, hoc eſt I H = F L; unde (ob parallelas rectas) erit I L = F H: Et (ſi A B dicatur axis) ob angulum I N L rectum, erit I L ⊏ L N, *i. e.* F H ⊏ L N; unde multo magis F H ⊏ D B, *i. e.* G H ⊏ E D.

2. Si A B non ſit axis ſed alia quælibet diameter, & M F quælibet diameter ab axe remotior quam A B; erit angulus quem faciunt rectæ A B, E D ad partes axi contrarias obtuſus, cum (per *Coroll.* 2. prop. 5.) ad partes axis ſit acutus; hoc eſt, erit angulus A B E, ideoque & huic æqualis L N I obtuſus; ergo in hoc quoque caſu L I ⊏ L N; cæteraque conſequentur ut in caſu priore, unde patet propoſitum.

Coroll. 1. Hinc (contra quàm in Ellipſi) majoris diametri tranſverſæ major eſt ſecunda huic conjugata, minoris minor. Nam quo magis diameter hyperbolæ ab axe diſtat eo major eſt, ut in *Cor.* 2. Prop. 2. oſtenſum eſt.

Coroll. 2. Poſitis ut prius, ſit contingentium HFG, EBD concurſus O, erit $FO \sqsubset BO$. Nam (per Prop. 16. p. 1.) $GFq : EBq :: OFq : OBq$; unde (ob $GF \sqsubset EB$) erit $OF \sqsubset OB$. Idem erit in Sectiones oppoſitas contingentibus.

Lemma.

25. 26. *In figuris* ABCD, *ſit* $BC = CD$, *& ang.* ACD *obtuſus,* i. e. *ang.* ACB *acutus, Dico* $AD \sqsubset AB$.

Ab A in DB (ſi opus productam) cadat perpendicularis AE, ob ang. acutum cadet hæc ad puncti C partes B, ideoque $DE \sqsubset BE$; Eſtque

$$ADq = AEq + DEq$$
$$ABq = AEq + BEq \sqsupset AEq + DEq$$

unde $AD \sqsubset AB$.

Prop. VIII. Theor. IV.

27. 28. *Parabolam in punctis* B, D *contingant* BA, DA *concurrentes in* A, *tactuſque* B *vel axis* LMN *vertici coincidat, vel ſit eidem axi tactu* D *propior,* i. e. *ordinata ad axem* BM *ſit minor quam* DN; *Dico* $DA \sqsubset BA$.

Connexâ DB & biſectâ in C, erit juncta AC (per Prop. 20. p. 1.) ejuſdem diameter, & parallela MN; Si B ſit axis vertex, vel ſi B, D ſint ex eadem parte axis, manifeſtum eſt AC non coincidere axi; Idem liquet ſi ſint ex axis partibus diverſis propter $BM \sqsupset DN$ & $CB = CD$: Producta ſi opus DB occurret axi in O, eritque ang. LOD *i. e.* ACD obtuſus, Unde per Lemma præcedens liquet propoſitum.

Prop. IX. Theor. V.

29. 30. 31. *In omni Sectione Conicâ, ad axem* AHI, *qui ſit determinatus Hyperbolæ, ſed utervis Ellipſeos, ordinetur recta* CHB, *occurrens Sectioni in* C, B; *In punctis vero* C, B *Sectionem contingant* CA, BA *ſuper*

axi concurrentes in A, *vel in Ellipſi forte axi parallelæ*; *deſcribaturque circulus* CEBI *qui rectam* CA (i. e. *ſectionem*) *in* C *contingat*, *tranſeatque per punctum* B: *Dico eundem rectam* BA, *vel* (*quod perinde eſt*) *ſectionem, in* B *contingere*; & *præter puncta* B, C, *totum eſſe intra ſectionem, ſi in Ellipſi* AHI *fuerit axis major*; *Sin* AHI *fuerit Ellipſeos axis minor, præter eadem puncta* C, B, *totum eſſe extra ſectionem.*

29. 30. 31. In omnibus caſibus occurrat axis ſectioni in D, circulo in E; Si concurrant CA, BA, propter axem erit (per *Coroll.*4. Prop. 2. & *Coroll.*1. Prop. 3.) AB = AC, unde AB (ex Elementis) circulum continget in B, id eſt (ex def.) circulus ſectionem continget. Per D & E agantur FD, GE parallelæ BC, utrivis contingenti BA occurrentes in F G: Cùm ſit axis AH ad BC perpendicularis, erit ſectionis & circuli communis diameter, ideoque (per *Coroll.* 8. Prop. 20. p. 1.) FD ſectionem, GE circulum continget, eritque (per *Coroll.* 3. Prop. 4. vel *Coroll.* 2. Prop. 7. vel per Prop. 8.) BF $\sqsubset$ FD, vel ſi AH ſit Ellipſeos axis minor FD $\sqsubset$ BF; Si ſit BF $\sqsubset$ FD, cum ſit propter circulum BG = GE, erit neceſſariò BG $\sqsupset$ BF; nam ſi BG dicatur $\sqsubset$ vel = BF, cadet GE ad rectæ FD partes A, vel eidem coincidet, hoc eſt erit GE $\sqsupset$ vel = FD, adeoque (ob BF $\sqsubset$ FD) erit BG $\sqsubset$ GE contra circuli naturam. Eſt ergo BF $\sqsubset$ BG *i. e.* (ob rectas parallelas) HD $\sqsubset$ HE, hoc eſt, punctum E cadet intra ſectionem. Pari ratione occurſus alter axis cum circulo I erit intra ellipſin: Nam in hyperbola & parabola per ſe manifeſtum eſt.

31. Si AHI ſit axis minor ellipſeos, erit BF $\sqsupset$ FD; unde haud abſimili modo oſtendetur puncta E, I cadere extra ſectionem. Propter duos contactus B, C, circulus (per Prop. 1.) ſectioni non amplius occurrit: ergo in priori caſu (*viz.* cum E, I ſint intra ſectionem) circulus totus erit intra ſectionem; in poſteriori (ubi hæc puncta ſunt extra ſectionem) totus erit extra ſectionem.

Si in Ellipſi A B, A C parallelæ ſunt, erit B C axis; nde (ex *Coroll.* 8. Prop. 2.) patet propoſitum.

Coroll. In figuris 29, 30. Liquet Circulum C E B I 29, 30. naximum eſſe omnium ſectioni inſcriptibilium & ean lem in puncto C tangentium; In fig. 31. Circulum 30, 31. EBI minimum eſſe omnium Ellipſi circumſcriptibi lium & eandem in puncto C tangentium: Nam (fig. 29, 0.) ejuſmodi major quivis ſecabit B C extra ſectionem; n fig. 31. minor quivis ſecabit B C intra ſectionem.

Prop. X. Theor. VI.

Iiſdem poſitis, rectiſque C A, B A *concurrentibus, ſi* 32, 33. *per* B *agatur diameter* B M K, *& ad hanc à puncto* C *Supple figuras hyperb. & parab.* *ordinetur recta* C V R, *occurrens ſectioni denuò in* R; *deſcribatur verò circulus rectam* C A (i. e. *ſectionem*) *in puncto* C *contingens, tranſienſque per punctum* R; *Dico hunc circulum præter tactum* C *in unico hoc puncto* R *ſectioni occurrere, Hoc eſt, ex unâ parte rectæ* C R *totum eſſe extra ſectionem, ex alterâ intra ſectionem.*

In Hyperbolâ, Parabolâ, & Ellipſi cujus axis major 32. t A H, Circulus rectam C A in C contingens & per unctum quodvis N ſectionis à B diverſum tranſiens, ajor erit circulo (in priore prop.) per B tranſeunte, nam circulus per B totus intra ſectionem cadit,) & ſe abit rectam C B productam extra ſectionem in S, hoc ſt, circulus C N S ad partes B erit extra ſectionem. Ex N ordinatâ ad diametrum B M K rectâ N T O, ccurret circulus ſectioni denuo in O: Nam producta N occurrat A C in L; & propter O N L $\parallel$ A B, rit (per Prop. 17, & 18. p. 1.) C L q : L N $\times$ L O : : A q : A B q *i. e.* (ob C A q $=$ A B q) erit C L q $=$ N $\times$ L O, unde cùm punctum N ſit ad circulum, erit O; & cum C ſit contactus, circulus C N S O (per rop. 1.) niſi in C, N, O ſectioni non occurret; hoc eſt, m S ſit extra ſectionem, arcus N O erit extra ſectionem, arcus verò CN, CO intra ſectionem: Nam ſi dicatur circulus ex utrâque parte N, vel O, eſſe extra ſectionem, ex tribus occurſibus C, N, O ſaltem duo erunt contactus, quod (per Prop. 1.) fieri nequit; eritque (ob punctum R extra circulum CNO) circulus CR major circulo CNO.

Augente ſe circulo CNO, & accedente puncto N ad C, accedet ſimul O ad R, & coincidentibus tandem N, C, coincident ſimul O, R; & evaneſcente arcu CN, totus arcus CSO, *i.e.* jam CR erit extra ſectionem; vel circulus ad rectæ CR partes B, eſt extra ſectionem.

33. Porro ex alterâ parte rectæ CR, ſumpto in ſectione puncto N, quod in Ellipſi non ſit punctum K, ductâque LNO, ut prius; circulus in C contingens & per N tranſiens oſtendetur (ut prius) tranſire per O; circulus vero CNO niſi in C,N,O ſectioni non occurrit; & in Hyperbolâ & Parabolâ (quæ ex hac parte infinitæ ſunt) manifeſtum eſt arcum NO eſſe intra ſectionem.

33. In Ellipſi vero ductâ contingente KP, erit hæc parallela AB; unde (ob ABq : ACq :: KPq : PCq & AB = AC) erit KP = PC, unde punctum P (per *Coroll.* 5. Prop. 2.) eſt ad axem minorem, & CK erit ad eundem axem ordinata, circuluſque contingens in C & per K tranſiens (per Prop. præced.) erit totus extra ſectionem, hoc eſt comprehendet punctum N vel O; unde (ob tactum C) circulus CNO minor erit circulo per K, & ſecabit rectam CK intra ſectionem in S, hoc eſt, arcus NSO erit intra ſectionem.

33. In omnibus igitur ſectionibus arcus NO eſt intra ſectionem: Erunt vero in omnibus, arcus CN, CO extra ſectionem; nam ſi dicatur circulus ex utrâque parte puncti N vel O eſſe intra ſectionem, è tribus occurſibus duo ſaltem erunt contactus, quod abſurdum eſt. Comprehendit inſuper circulus CNO punctum R, proindeque eſt (ob tactum C) circulo CR major.

Minuente ſe circulo CNO, & accedente puncto N ad C, accedet ſimul O ad R, & coincidentibus N, C, coincident ſimul O, R; & evaneſcente arcu NC, totus

arcu

rcus C S O, hoc eſt, jam arcus C R erit intra ſectionem;
vel circulus ad rectæ C R partes K (*i. e.* ipſi B contra-
ias) eſt intra ſectionem. Liquet ergo propoſitum.

Coroll. 1. Hinc; Quámvis circulus Sectioni Conicæ
ccurrens, & ex utrâque occursûs parte ad eaſdem ſe-
tionis partes cadens, in eodem occurſu ſectionem ne-
ceſſariò contingat; Non tamen circulus ſectionem co-
nicam contingens neceſſariò ex utrâque contactûs par-
te ad eaſdem ejus partes cadit.

Coroll. 2. Cum omnis circulus ſectionem in C con- 32.
ingens, & per punctum quodvis ſectionis ex unâ parte
rectæ C R (quantumvis puncto R propinquum) tranſiens
minor ſit circulo C R, & ex utrâque parte contactûs
intra ſectionem reperiatur, hoc eſt, magis incurvetur 33.
quàm ſectio in puncto C; Circulus vero illam in eo-
dem puncto contingens & per punctum quodvis ſectio-
nis ex alterâ ejuſdem rectæ parte tranſiens ſit major
circulo C R, & ex utrâque contactûs parte cadat extra
ſectionem, hoc eſt, minùs incurvetur quàm ſectio in
eodem puncto C; Sectionem in hoc puncto C circulo
C R æquicurvam eſſe, meritò exiſtimare licet.

Schol. Si axis A H in Ellipſi ponatur minor, eadem
erit demonſtratio ſed inverſa; *viz.* pro *major*, legen-
do *minor*, pro *intra*, *extra*; & vice verſâ.

Prop. XI. Theor. VII.

Iiſdem poſitis; per C *ducatur diameter* CL, *quæ* (*ſi opus* 34. 35.
producta) *occurrat circulo* C R *in* Q; *Dico* C Q *æqua-* *Supple figu-*
lem eſſe rectæ D F *quæ eſt parameter diametri* C L. *ras parab. & hyperb.*

In omnibus ſectionibus, ſit Circulus C N O idem
qui in figuris præcedentibus, ſecans C Q in X; à pun-
cto N (*viz.* punctorum N, O rectæ A C propinquiore)
ad diam. C L ordinetur N P I S, occurrens ſcilicet dia-
metro C L in I, & circulo denuò in S; & à puncto C
erigatur ad C A perpendicularis C P, ſecans N S in P;
Erit hæc circuli diameter, & ob C A || N S) biſecabit

NS in P. Si punctum I non ſit primo intra circulum CNO; Augente vel minuente ſe circulo CNO prout puncta N, O fuerint ad has vel illas partes rectæ CR, cùm, accedente N ad C, & O ad R, accedat continuò I ad C, & X ad Q, tranſibit tandem neceſſariò punctum I intra circulum CNO; eritque PI (ob $SP = PN$) ipſarum IS, IN ſemidifferentia.

In Hyperbolâ & Ellipſi, in rectâ DF (productâ in hyperbolâ) intelligatur punctum E mobile, hâc lege, ut accedente I ad C, ſit ſemper $LC : CI :: DF : FE$; Eritque (per Prop. 24. p. 1.) $CI \times DE = NIq$, hoc eſt,

$CI : NI :: NI : DE$; & ob circulum erit

$CI \times IX = NI \times IS$, hoc eſt,

$CI : NI :: IS : IX$; Unde

$NI : IS :: DE : IX$.

Ex alterâ parte CL, eſt continuo $NI \sqsubset IS$, adeoque $DE \sqsubset IX$, ex alterâ $IS \sqsubset NI$, *i. e.* $IX \sqsubset DE$; utrobique vero accedente N ad C, accedit ſimul O ad R, X ad Q, & I ad C, fitque PI continuò minor, hoc eſt NI, IS adeoque DE, IX continuò propius ad æqualitatem accedunt; & tandem coincidentibus S, I, P, N, ipſi C; O ipſi R; adeoque X ipſi Q, & evaneſcente PI, fit IX *i. e.* jam $CQ = DE$; evaneſcente verò jam CI evaneſcit ſimul FE, *i. e.* $CQ = DF$.

In Parabolâ, Erit (omiſſo puncto E) propter ſectionem $CI \times DF = NIq$, hoc eſt,

$CI : NI :: NI : DF$, & propter circulum

$CI : NI :: IS : IX$; unde

$NI : IS :: DF : IX$; Et ad morem præcedentium, oſtendetur, coincidentibus punctis, ipſas DF, IX ſimul fieri ſibi mutuò, & rectæ CQ, æquales.

Corollaria ad hanc & prop. præced.

34, 35, 36, 37, 38. *Coroll.* 1. In Sectionis Conicæ quâlibet diametro CL (productâ ſi opus in Ellipſi,) ſi fiat CQ æqualis ejuſdem diametri parametro, & in puncto C ſectionem contingat recta CA; Circulus rectam CA in C contingens &

& per punctum Q tranſiens erit ſectioni in hoc puncto C æquicurvus. Si diameter C L non ſit axis, ſatis liquet ex præcedentibus, nam non erit à circulo CR diverſus. Cùm vero hæc proprietas competat cuilibet diametro præter axem, etiam axi competet; Nam diameter axi infinite vicina ab axe non differt, neque hujus parameter ab axis parametro.

Coroll. 2. Coincidente puncto C axis vertici G, (qui 32.
in ellipſi ſit major,) coincident etiam ei puncta B, R, 36, &
arcuſque circuli C R qui prius erat extra ſectionem eva- 37.
neſcet, hoc eſt totus circulus C R erit intra ſectionem.
Puncto vero C (in Ellipſi) minoris axis vertici coinci- 33, & 38.
dente, arcus circuli qui prius erat intra ſectionem evaneſcet, hoc eſt, totus circulus C R erit extra ſectionem.

Coroll. 3. Arctior igitur ſive intimior eſt hujuſmodi 32, 33.
circuli & ſectionis conicæ contactus quàm ſimplex qui- & 36,
vis contactus, ſed in axis vertice arctiſſimus: Nam ſi 37, 38.
punctum C ſit extra axis verticem ſimplici contactui C circuli C N O coincidit occurſus N; In axis vero vertice his accedit & O, *i. e.* R.

Coroll. 4. In vertice axis parabolæ, determinati hy- 36, 37.
perbolæ, vel majoris ellipſeos, liquet circulum G Q *i. e.* C Q maximum eſſe omnium ſectioni inſcriptibilium & eandem in eodem vertice G tangentium. Nam in vertice diametri axi infinite vicinæ, quæquè ideo ab axe non differt, circuli æquicurvi arcus extra ſectionem erit infinite parvus, hoc eſt non differet circulus ab ipſo circulo G Q; & quivis ejuſmodi circulus circulo æquicurvo major ex utraque contactûs parte erit extra ſectio-
nem. Pari ratione in ellipſi erit Circulus G Q mini- 38.
mus omnium ſectioni circumſcriptibilium & eandem in vertice minoris axis G tangentium.

Coroll. 5. Liquet etiam in hoc caſu rectam C Q *i. e.* 36, 37,
G Q eſſe circuli diametrum. Unde ſumptâ in axe G Q 38.
= parametro axis, circulus diametro G Q erit ſectioni in axis vertice G æquicurvus.

Prop. XII. Theor. VIII.

39, 40. *Parabolam contingat utcunque* T D *in* T; *huic vero parallela* F E *occurrat ſectioni in* F, E *punctis, ex quibus ad quamlibet diametrum* D A O H *ordinentur* E G, F H, *& ex tactu* T *recta* T O; *Si puncta* E, F *ſint ex eadem parte diametri, dico duplum ordinatæ* T O *æquale eſſe ſummæ ordinatarum* F H, E G; *ſi ex partibus diverſis, earum differentiæ.*

Ductâ diametro T I M, biſecat hæc E F in I; Acta E L parallela T M occurrat T O, F H (ſi opus productis) in N, L. Ob parallelas I M, E L & æquales F I, I E, æquales erunt F M, L M, T N; ſed & (ob parallelas) æquales ſunt L H, N O, E G, ergo $FM \pm EG = LM \pm LH = TO$ *i. e.* $FH \pm EG = 2\,TO$.

Prop. XIII. Theor. IX.

41, 42, 43, 44, 45. *Si Circulus occurrat Parabolæ in quatuor punctis* A, B, E, D, Fig. 41, 42; *Vel in tribus* A, B, Q, *quorum* Q *ſit contactus*, Fig. 43, 44; *Vel in duobus tantum* Q, B, *quorum utrumvis* Q *ſit contactus, & reliquum* B *interſectio*, Fig. 45; *Sintque ab his occurſibus ordinatæ ad Parabolæ axem*, viz. A G, B I, E F, D H, Fig. 41, 42. A G, B I, Q O, Fig. 43, 44. *&* B I, Q O, Fig. 45. *Dico ordinatas ex unâ parte axis ſimul ſumptas æquales eſſe ordinatis ex altera parte ſimul ſumptis; Ordinata vero ex contactu, ubi tres ſunt occurſus*, Fig. 43, 44. *bis ſumpta; ubi duo tantum*, Fig. 45. *ter ſumpta intelligatur.*

41, 42. Si quatuor ſint occurſus; duos quoſlibet A, B jungat A B, & reliquos duos D E occurrens A B ſi opus productæ in N: Inventis rectarum B A, D E diametris P V, Q S, in earum verticibus P, Q ſectionem contingant P R, Q R concurrentes in R; erunt hæ rectis A B, D E paral-

parallelæ, eritque (per prop. 18. p. 1.) $RQq : RPq :: NE \times ND : NA \times NB$; ſed (propter circulum) $NE \times ND = NA \times NB$, ergo & $RQq = RPq$ *i. e.* $RQ = RP$. Connexâ PQ & biſectâ in O, erit junctâ RO ipſius diameter, quæ (ob $RP = RQ$) erit axis. Sed (per prop. præced.) in fig. 41. $BI + AG = 2PO = 2QO = FE + HD$; & fig. 42. $BI - AG = 2PO = 2QO = FE + HD$, unde $BI = FE + HD + AG$.

2. Si tres ſint occurſus quorum unus eſt contactus: 43, 44.
Intelligantur puncta E, Q, D (quæ in prioribus figuris diverſa ſunt) coincidere; Coincidentque rectæ DEN, QR; ſimiliter EF, QO, DH: Eritque fig. 43. $BI + AG = 2PO = 2QO$, *i. e.* $EF + DH$; Vel (ſi AG ſit ex eâdem parte axis quâ QO, fig, 44.) erit $BI - AG = 2PO = 2QO$, unde $BI = 2QO + AG$ *i. e.* $EF + DH + AG$.

In caſibus præcedentibus, ſi occurſuum aliquis A axis vertici coincidat, evaneſcente AG, fit unica ordinata ex unâ parte axis æqualis duabus ex parte alterâ, vel uni ordinatæ ex contactu bis ſumptæ.

3. Si duo tantum ſint occurſus, quorum unus Q eſt 45.
contactus & reliquus B interſectio (ut in caſu prop. 10.) Coincident puncta A, E, Q, D, N; Eſtque (per præced.) $BI - QO = 2PO = 2QO$, unde $BI = 3QO$ *i. e.* $AG + EF + DH$.

Prop. XIV. Theor. X.

In Parabolâ, cujus axis DAOB, *ejus vertex* A, *&* 46.
quælibet contingens TD, *occurrens axi in* D, *Sit* TO *ordinata à puncto* T *ad axim, &* TB *perpendicularis ad contingentem occurrens axi in* B; *Dico* OB *æqualem eſſe ſemiparametro axis.*

Nam (ob TO perpendicularem ex angulo recto) erit $TOq = BO \times OD =$ (per prop. 2. p. 2.) $BO \times 2OA =$ (per prop. 25. p. 1.) $OA \times$ param.

&

& (horum dimidia) $BO \times OA = OA \times \frac{1}{2}$ param. Unde $BO = \frac{1}{2}$ param.

Prop. XV. Theor. XI.

47. *In Parabola, cujus axis* BC, *ejus vertex* B, *alia quævis diameter* EF, *&* *ex* E *ordinata ad axem recta* EC; *dico* LR *parametrum diametri* EF *ſuperare* lr *parametrum axis quantitate quadruplâ rectæ* BC; i, e. $lr + 4BC = LR$.

In E contingat EA occurrens axi in A, & ex B ad diametrum EF ordinetur BF; erit BF parallela EA; unde (ob BC parall. EF) erit $FE = BA =$ (per prop. 2. p. 2.) BC, & $AE = BF$; eſtque (propter axem) angulus ACE rectus, ergo $lr \times BC$ (per prop. 25. p. 1.) $= CEq = AEq - ACq =$ (ob $AB = BC$) $AEq - 4BCq = BFq - 4BCq = LR \times EF - 4BCq = LR \times BC - 4BCq$; unde (propter æqualem altitudinem BC) erit $lr = LR - 4BC$ vel $lr + 4BC = LR$.

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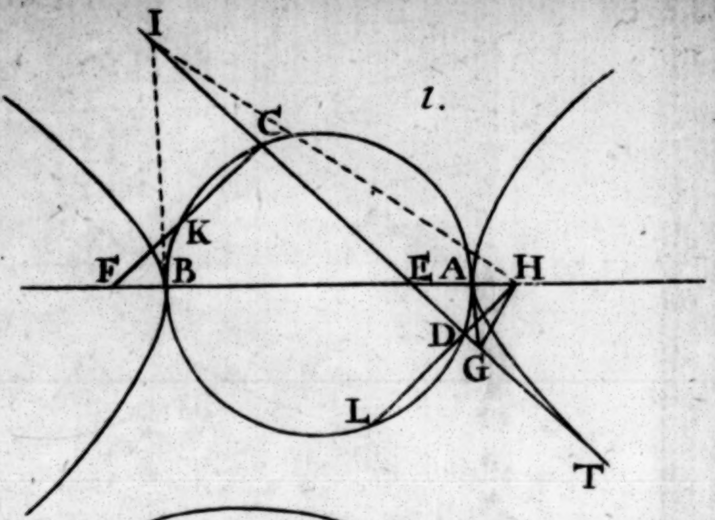
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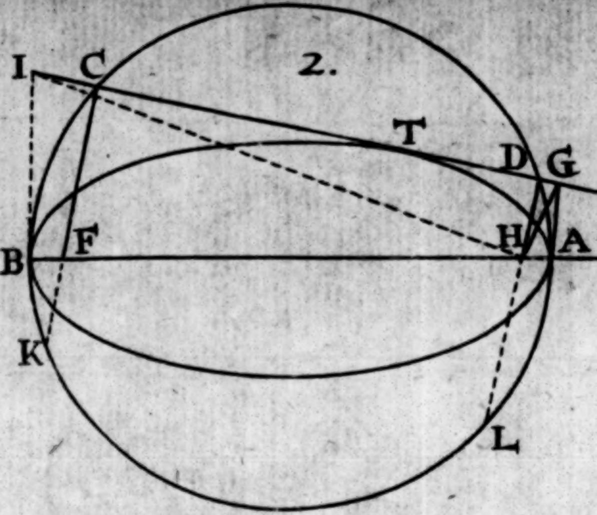
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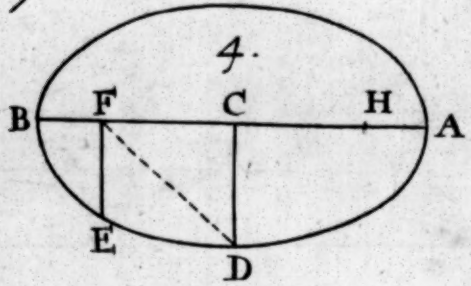
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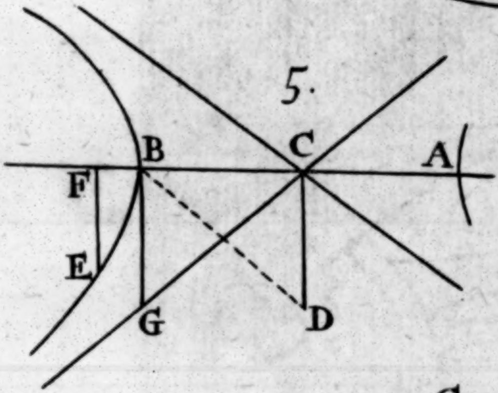
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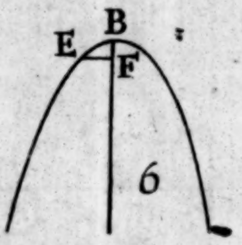
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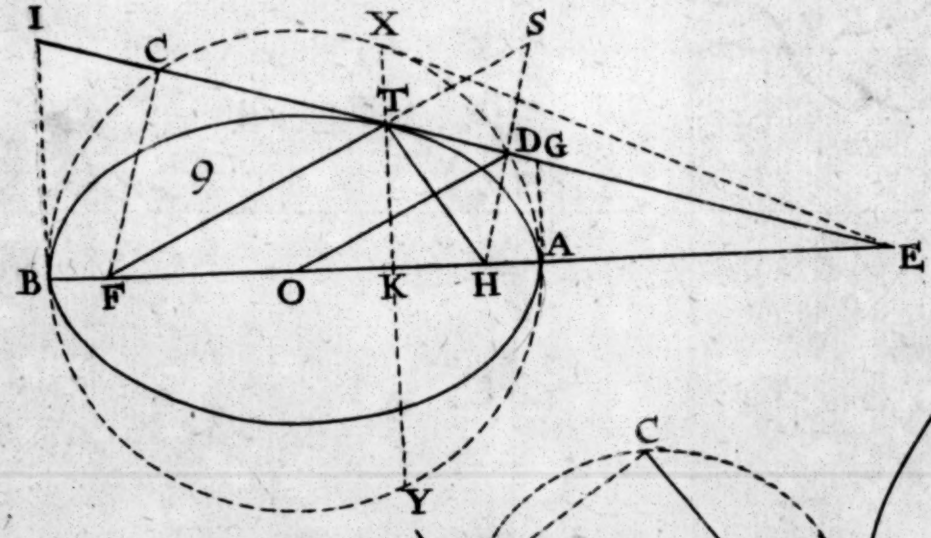
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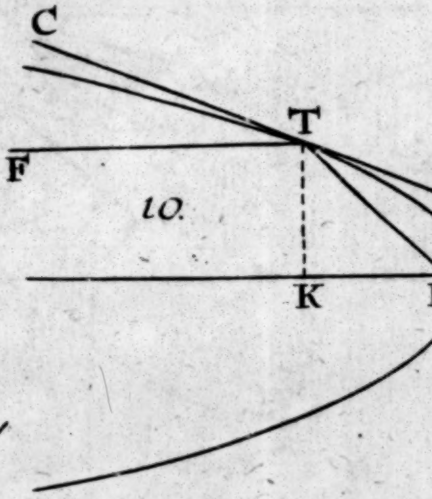
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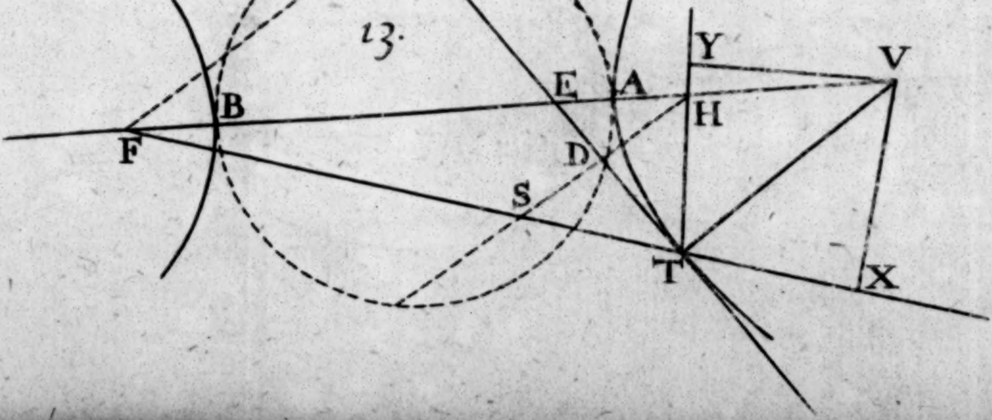
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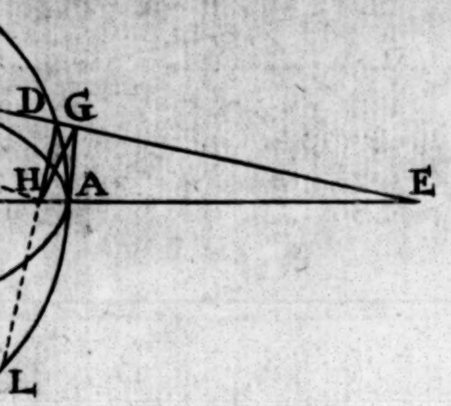
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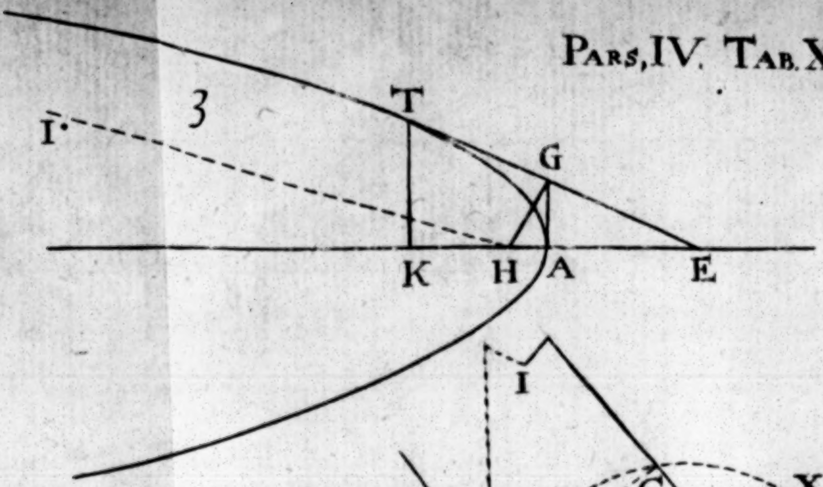
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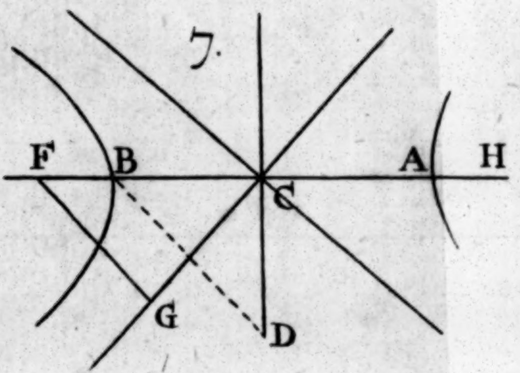
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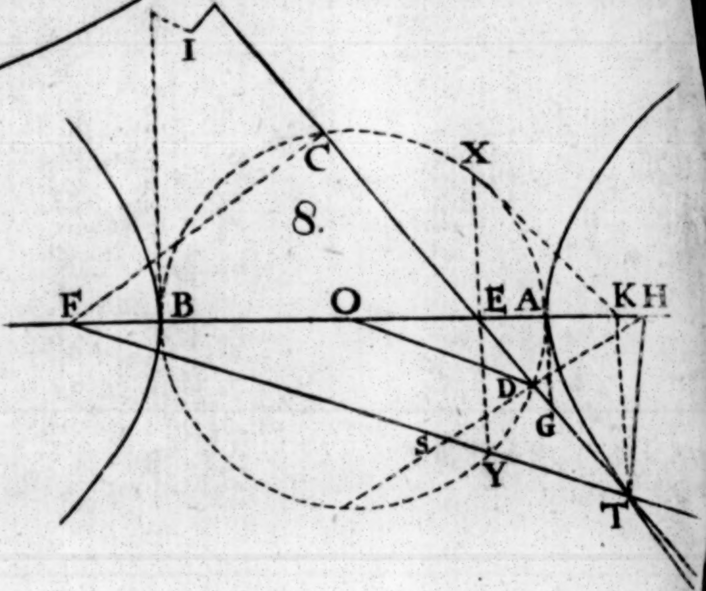
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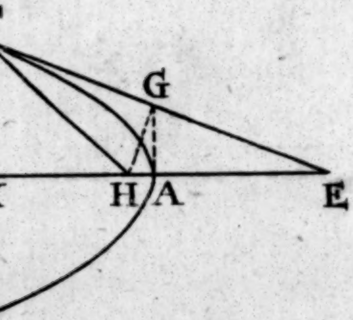
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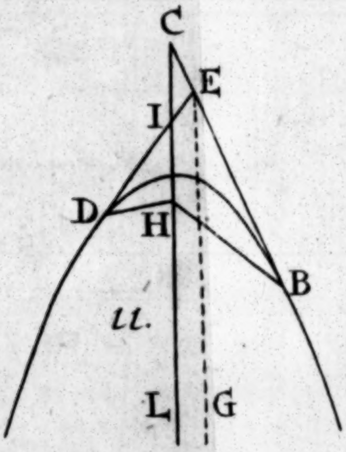
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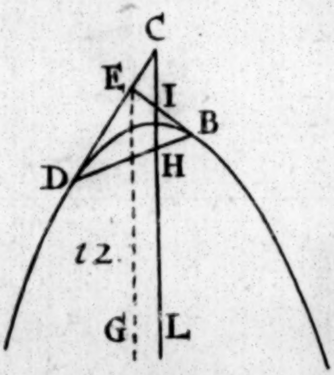
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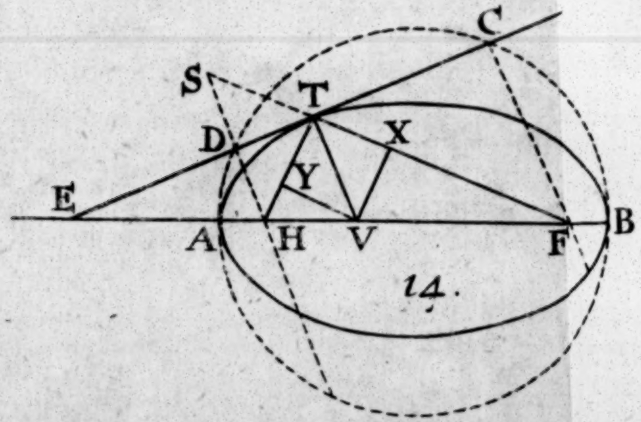
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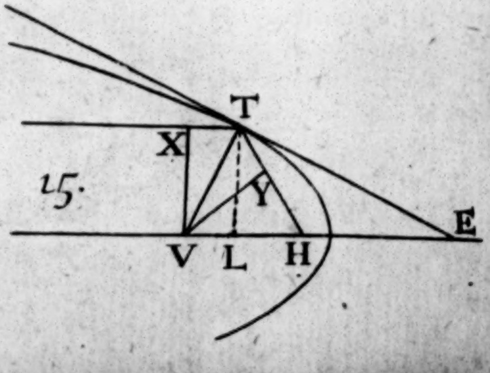
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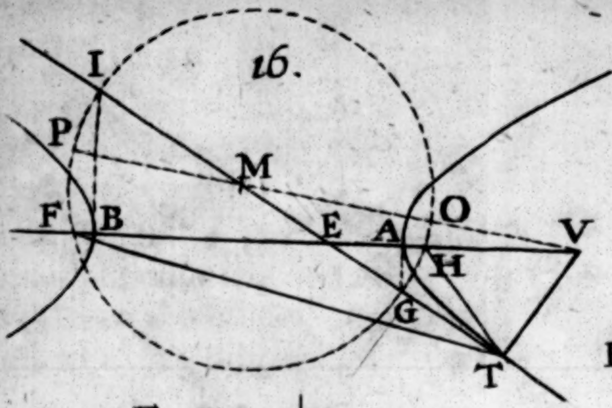
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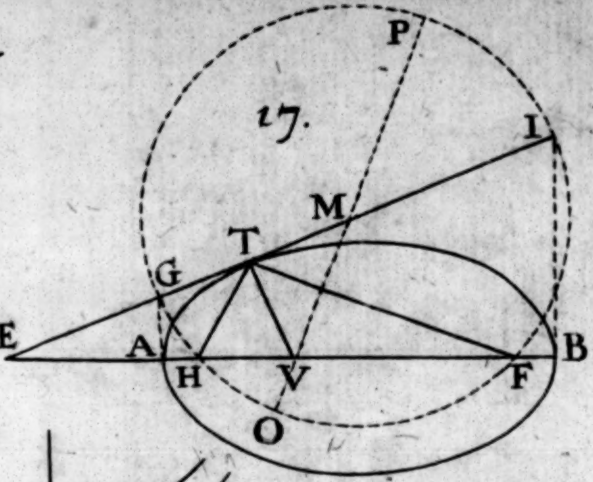
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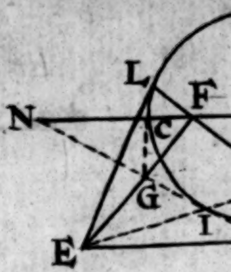

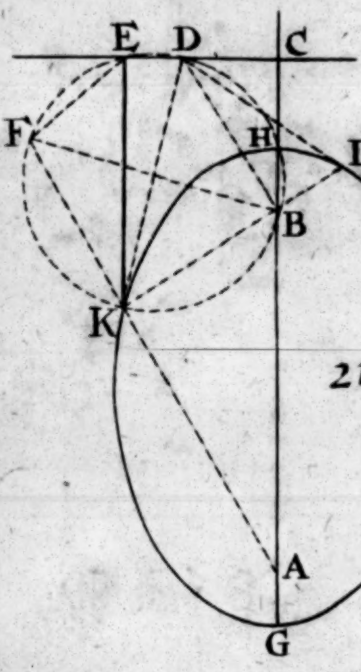
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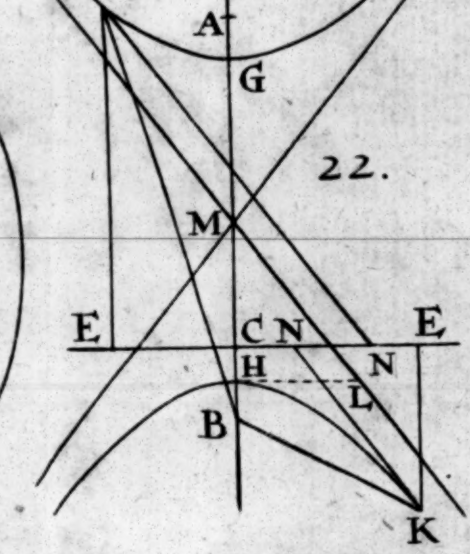
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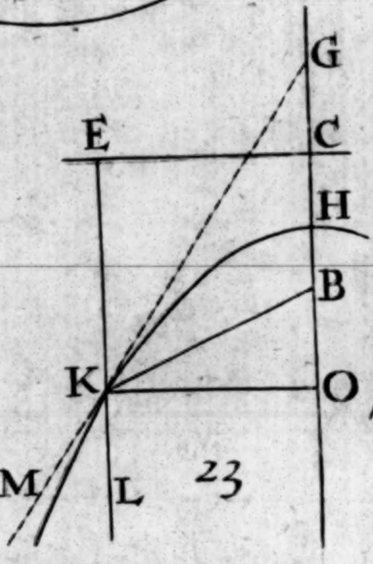
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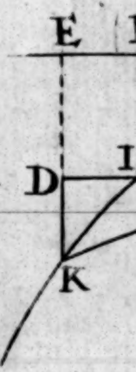

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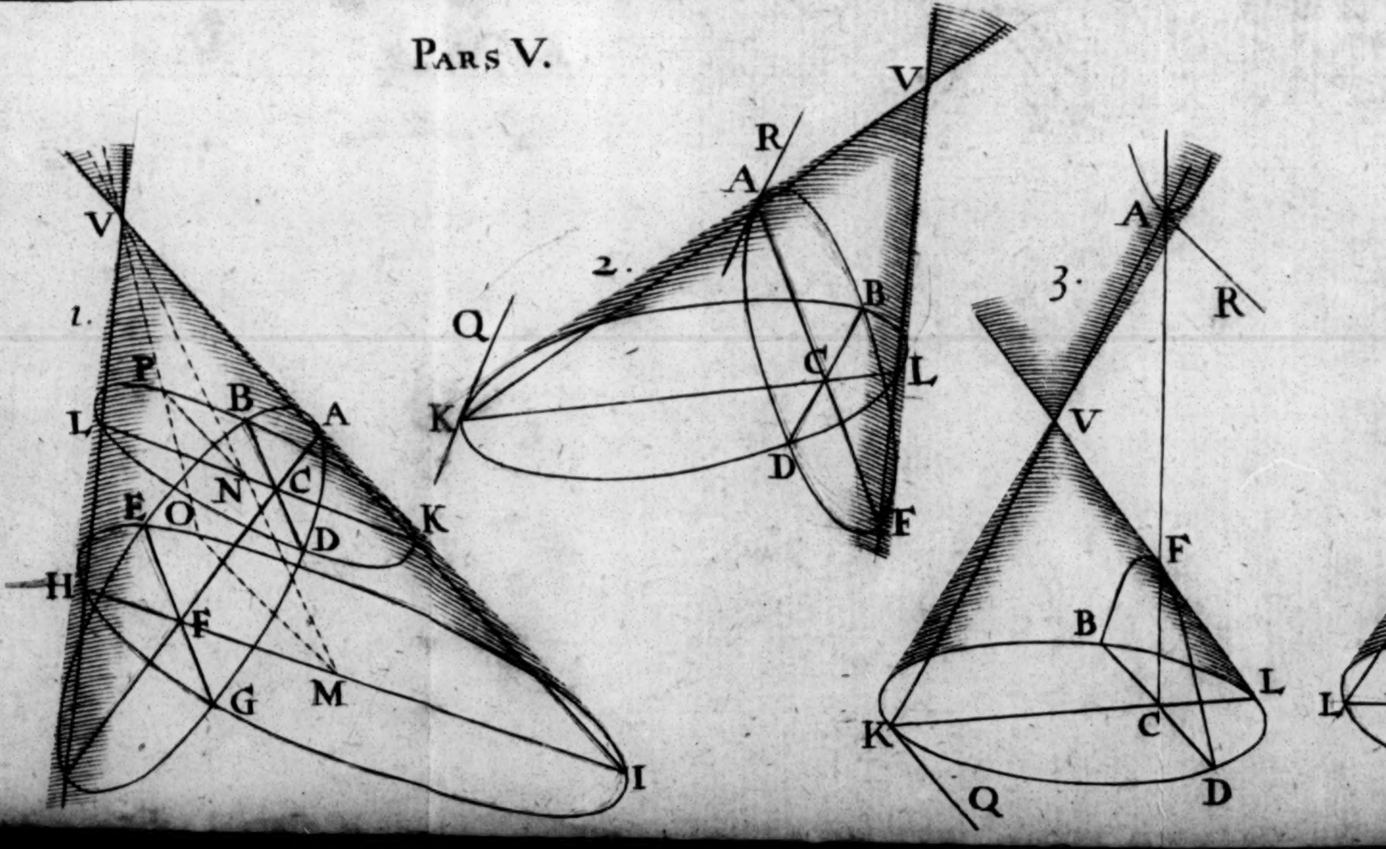
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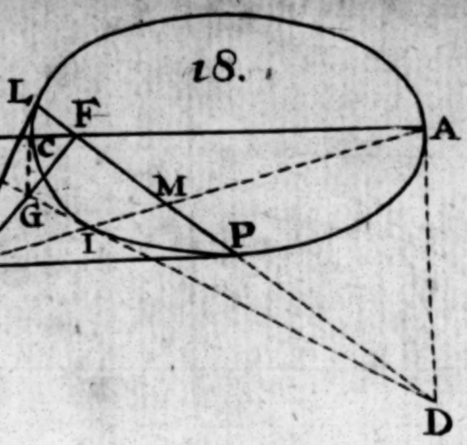
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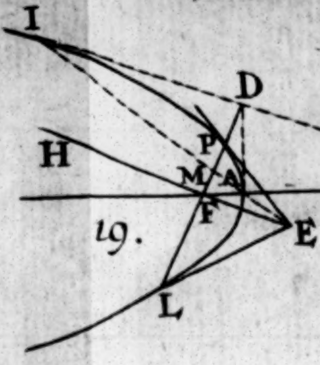
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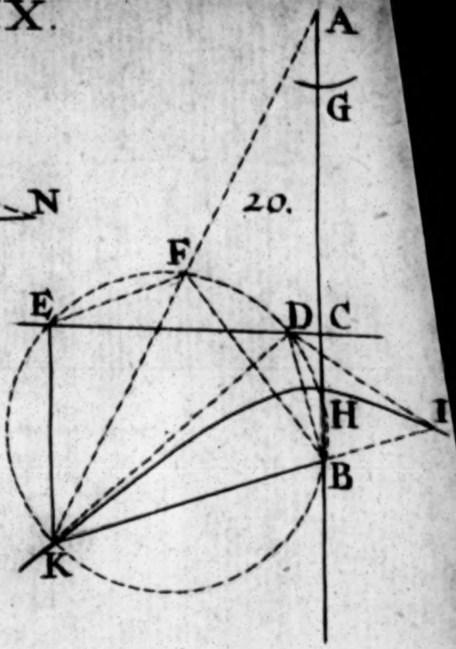
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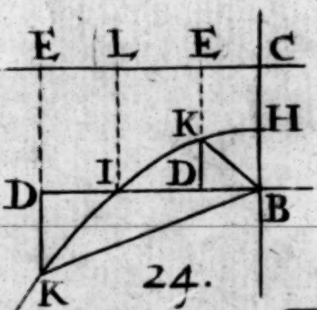
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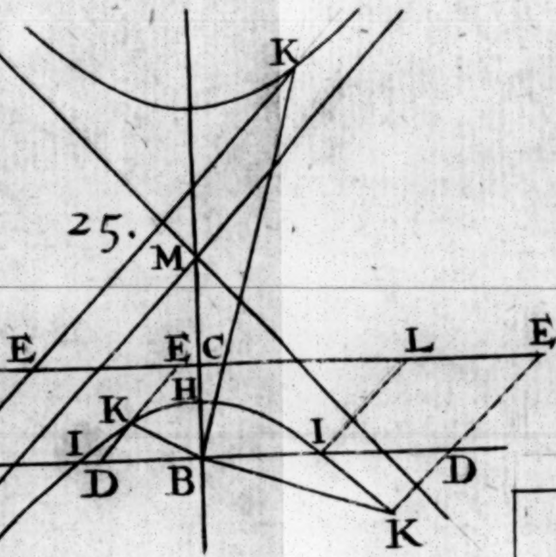
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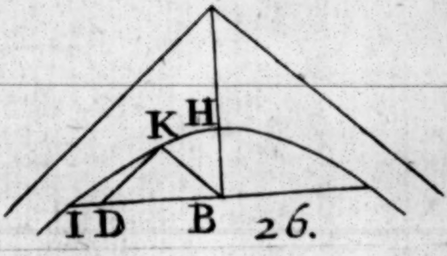
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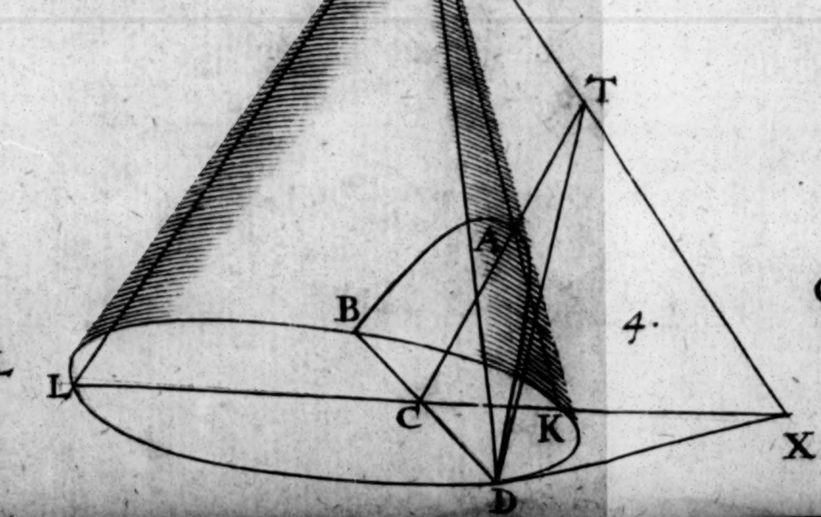
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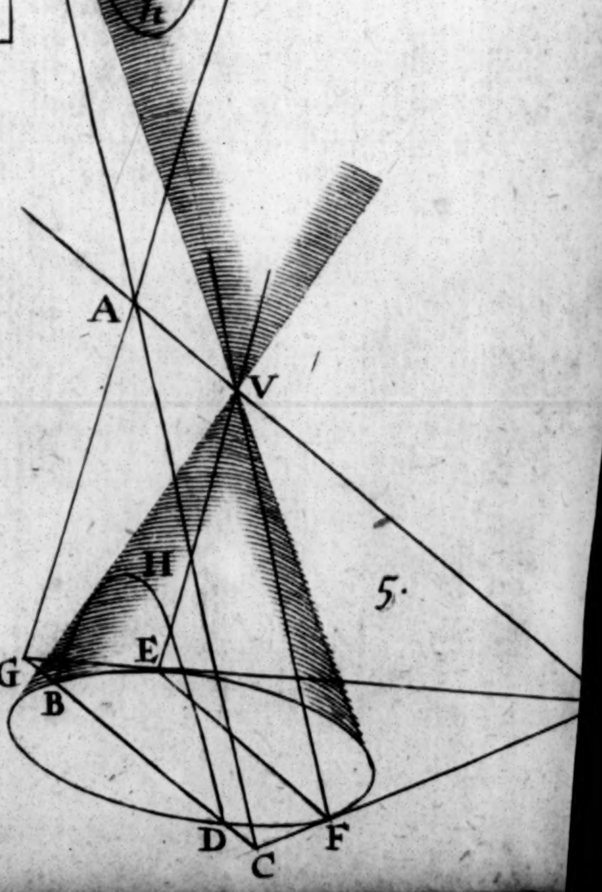
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Coroll.
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Coroll. Si BK ſit alteri aſymptoto parallela, erit tri- 26.
g. BKD iſoſceles; & ob $BK + KD = BI$ & BK
KD, erit $BK = \frac{1}{2} BI = \frac{1}{4}$ parametri axis.

Scholium. Inter Parabolæ diametros & rectas hy-
rbolæ [vel ſect. opp.] aſymptoto parallallelas, maxi-
a eſt (ut paſſim obſervare licuit) affinitas atque
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ſignia præ aliis exempla.

PARS

PARS V.

Propoſitio I. Probl. I.

1. *In plano quovis* ACD, *datâ poſitione rectâ* ACF *pro diametro ſectionis conicæ* [*vel ſect. opp.*] *quæ in hyp. vel. ſect. opp. ſit determinata, cum utrovis, vel unico, ejus vertice* A, *& duabus ordinatis* DC, GF, *ad eandem in quovis angulo* ACD *inclinatis*;

2, 3. *Vel datâ* (*ut priùs*) *diametro* ACF, *cum utroque vertice* A, F, *& unicâ ordinatâ* CD, *ad eandem in angulo* ACD *inclinatâ*;

4. *Vel denique diametro* AC, *cum uno vertice* A, *& unicâ ordinatâ* CD, *ad diametrum* AC *ſub angulo* ACD *inclinatâ, & rectâ* DT *pro contingente in ejus extremo* D;

Hujuſmodi ſectionem in ſuperficie conicâ exhibere.

Oportet autem nec puncta G, D, *nec rectam* DT, *cum dato vertice* A *in directum jacere.*

1. 1. PRoductis DC, GF in B, E, ut ſit CB = CD, EF = FG, Per parallelas BD, EG tranſeant duo plana invicem parallela BCK, EFI, ad planum ACD quomodolibet inclinata; In quorum utrovis, *ex. gr.* in illo per BCD, ſuper BD tanquàm chordâ ſtatuatur circulus BKDL; ex C erectâ in eodem plano ad BD perpendiculari LCK occurrente circulo in L, K, per rectas ACF, LCK tranſeat planum ſecans alterum ex planis parallelis in FI, quæ proinde erit ad LCK parallela.

Connexâ AK vel AL *ex gr.* AK, hæc (producta ſi opus) ſecet FI in I; deinde in plano EFI per puncta E, G, I deſcribatur circulus ſecans IF in H; connexa denique HL ſecet AK (utpote in eodem plano ſitam) in V.

Superficiei conicæ, vertice V, baſi circulo BKDL vel HEIG, genitæ, cum plano ACD interſectio, *viz.* EBADG, eſt ſectio propoſita.

Ob rectas BD, EG, & LK, HI parallelas, & ang. BCK (ex conſtr.) rectum, erunt LK, HI circulorum diametri. Biſectâ LK in N, ductâque VN (& ſi opus productâ,) biſecabit hæc HI in M, eritque VNM coni axis utrâvis baſi, vertice V geniti.

Per VNM tranſeat planum ſecans circuli LBKD planum in NP, planum vero circuli HEIG in MO, eorumque peripherias in P, O, & connectantur PV, OP.

Erunt (ob plana parallela) PN, OM parallelæ, & propterea quòd N, M ſint circulorum centra, erit NK = NP, MI = MO, & ob ſim. triang.

$$VN : VM :: \left\{\begin{matrix}NK\\NP\end{matrix}\right\} : \left\{\begin{matrix}MI\\MO\end{matrix}\right\}$$ unde rectæ VP, PO in directum jacent, & ſic ubique; Una itaque eademque eſt ſuperficies conica ſive baſi LBKD, ſive HEIG, & vertice V genita. Tranſit verò per puncta G, D, A, B, E, biſecatque ACF rectas BD, EG, in C, F, unde (per *Coroll.* 10. prop. 20. p. 1.) eſt harum diameter; ideoque ſectio GDABE eſt ſectio propoſita.

2, 3.

2. Si detur unica ordinata CD cum utroque vertice A, F: Factâ CB = CD, ductoque circulo LBKD, & rectâ LCK, ut in priore caſu; In plano per ACF, LCK connexa LF occurrat connexæ AK in V, eritque ſuperficiei conicæ vertice V, baſi LBKD, genitæ, cum plano ACD interſectio, *viz.* FBAD ſectio propoſita.

Tranſit enim per puncta B, A, D, F; Si vero planum ſecundùm rectam VK ſuperficiem contingat, ſecans planum circuli in KQ, planum vero ſectionis in AR, erit ang. CKQ rectus, adeoque KQ parallela CD, quæ (per lemma 1. p. 1.) eſt parallela AR; tangit verò A R ſectionem in A, biſecatque A C rectam B D in C, unde (per *coroll.* 11. prop. 20.) eſt rectæ B C D atque huic parallelarum diameter; ideóque F B A D ſectio propoſita.

4. 3. Si, dato uno tantum vertice A, detur unica ordinata D C, cum rectâ D T pro contingente in ejus extremo: Ducto circulo L B K D & rectâ L C K, ut priùs, circulum contingat D X in D; Per D T & D X tranſeat planum ſecans planum per A C, L C K in rectâ X T (niſi forte fuerit planum T D X plano A C K parallelum) cui occurrat connexa A K in V; Erit ſuperficiei conicæ vertice V, baſi L B K D, genitæ, cum plano A C D interſectio, *viz.* B A D, ſectio propoſita.

Tranſit enim per B, A, D; Et quoniam, connexâ V D, planum V D X tangit ſuperficiem, recta D T (quæ eſt in plano contingente) tanget ſectionem: Quòd verò recta A C ſit ordinatæ B C D, atque huic parallelarum, diameter, patet ut in priore caſu.

Supple hos caſus.

Si in 1^{mo} vel 3^{tio} caſu datus unus vertex A, in 2^{do} è duobus alteruter A, infinitè diſtet, (non exiſtente in 1^{mo} connexâ G D, in 3^{tio} rectâ D T parallelâ A C) hoc eſt, ſi reliquis datis, exhibenda ſit ſectio quæ ſit Parabola; Ductâ rectâ A K parallelâ A C F, manet reliqua conſtructio: Erit verò (ob A K parallelam A C) planum ſectionis plano ſecundùm V A K ſuperficiem tangenti parallelum, ideoque ſectio erit Parabola. In cæteris demonſtratio haud erit diverſa.

Si punctum V infinitè diſtet, hoc eſt, ſi rectæ H L, A K in primo caſu, F L, A K in ſecundo, vel in tertio K A, T X, parallelæ ſint, fit ſuperficies cylindrica, quæ eſt ſuperficies conica cujus vertex infinitè diſtat: ſed mutatâ planorum ad invicem inclinatione, manente circulo; vel aſſumpto majoris minoriſve diametri circulo, manente planorum inclinatione, vertex V ad diſtantiam finitam accedet. Idem intellige ſi in tertio caſu planum T D X ſit plano A C K parallelum.

Si puncta G, D in primo caſu, vel recta D T in tertio,

tio, & datus vertex A in directum jacere intelligantur, Coincident puncta A, V, & loco ſectionis conicæ prodibunt duæ rectæ, *i. e.* ſectio per verticem.

Si ſimul datus vertex A infinitè diſtet, & vel connexa G D in primo caſu, vel recta D T in tertio, ſit datæ diametro parallela, abeunte coni vertice in infinitum, Parabola in duas rectas diametro parallelas, & ab eâdem utrinque æqualiter diſtantes abibit.

Coroll. Cùm in quovis hujus propoſitionis caſu, tam 1, 2,
circulus L B K D, modò recta B D eidem inſcribi poſſit, 3, 4.
quàm plani ejuſdem circuli ad planum A C D inclinatio pro libitu aſſumatur; His quomodolibet variatis, orientur ſuperficies conicæ numero infinitæ, quarum quævis propoſito ſatisfacit.

Prop. II. Probl. II.

Datis in plano duabus rectis non parallelis A G, A C, 5.
pro Hyperbolæ [*vel ſect. opp.*] *aſymptotis, & extra eaſdem puncto* B *per quod ſectio tranſeat, Hujuſmodi ſectionem in ſuperficie conicâ exhibere.*

Per B ductâ G B C datas rectas in G, C ſecante, factâque $CD = BG$, ſuper B D tanquam chordâ ſtatuatur cujuſvis magnitudinis Circulus B E F D, in plano ſcilicet ad planum A G C quomodolibet inclinato: Ex punctis G, C circulum contingant G E, C F; junctiſque tactibus E, F, erit E F parallela G C: Nam (ob $GB = DC$, & propter circulum) $GD \times GB = GEq = CD \times CB = CFq$, *i. e.* $CF = GE$; unde, ſi G E, C F parallelæ ſint, erit E F parallela G C; ſin concurrant in I, erit (ob circ.) etiam $EI = FI$, unde iterùm erit E F parallela G C.

Per E F igitur tranſeat planum plano A G C parallelum, ſecans plana per A C, C F & A G, G E in rectis F V, E V: Si vertice V, baſi circulo B E F D, generetur conica ſuperficies, erit ejus interſectio cum plano G A C, *viz.* B H D, ſectio propoſita.

Nam, cùm ſectio fiat à plano G A C plano per verticem E V F ſuperficiem ſecante parallelo, erit Hyperbola [vel ſect. opp.] Tranſit etiam per B; Tangunt vero plana V F C, V E G ſuperficiem conicam; proindeque erunt A G, A C (per prop. 13. p. 1.) ſectionis aſymptoti.

5. *Coroll.* Pro variâ circuli B E F D magnitudine, plani ejuſdem circuli ad planum G A C inclinatione, vel rectæ G B C poſitione, orientur ſuperficies conicæ numero infinitæ, quarum quævis propoſito ſatisfacit.

Prop. III. Probl. III.

6, 7, 8. *Datis in plano quinque punctis* P, Q, R, S, T, (*quorum unum, vel duo quævis, ſecundum datas directiones à reliquis infinite diſtare poſſunt,*) *per quæ ſectionem conicam* [*vel ſect. opp.*] *tranſire oportet*; *Hujuſmodi ſectionem in ſuperficie conicâ exhibere.*

Oportet autem ex datis punctis nulla tria in directum jacere; *Nec* (*quod idem eſſet*) *punctum aliquod ſecundum rectæ duo alia conjungentis directionem infinite diſtare.*

Connexis *ex. gr.* Q P, R P, [aut ductis ſecundùm datam directionem parallelis,] agantur his reſpectivè parallelæ T X, T Y, quibus connexæ Q S, R S, [aut ductæ ſecundùm datam directionem ad invicem parallelæ] occurrant in x, y, reſpectivè; junctâque R Q, ſecante T X in X, fiat T x : T X : : T y : T Y, (ſumpto Y ad eaſdem partes puncti T reſpectu y, ad quas jacet X reſpectu x,) & connectatur R Y.

Per Q ducatur Q K ipſi R Y parallela, ſecans T X in K, fiatque T x : T y : : T K : T I, (ſumpto I ad eaſdem partes puncti T reſpectu y, ad quas jacet K reſpectu x,) & connectatur R I occurrens Q K in V.

Biſectâ Q V in M, connexâque R M, per T agatur T O ipſis R Y, Q V parallela, occurrens R M in O; Si diametro R O M, vertice R, ordinatis Q M, T O, fiat in ſuperficie conicâ (juxta prop. 1.) ſectio conica [vel ſect. opp.] erit eadem ſectio quæſita.

Nam recta R Y (per *coroll.* 8. prop. 20. p.1.) ſectionem in R contingit; Suntque (per conſtr.) præter tactum R puncta T & Q ad ſectionem, adeoque (ob Q M = M V) punctum V; Unde, propter $Tx : Ty :: TX : TY :: TK : TI$, ſectio (per *cor.*3. prop. 35. p.1.) tranſibit per P; & propter $TX : TY$ vel $TK : TI :: Tx : Ty$ (per ejuſdem prop. *coroll.* 2.) tranſit per S; aut ſaltem erunt Q P, R P, vel Q S, R S ſectionis aſymptoto vel parabolæ diametris parallelæ.

Incidente puncto V in Q, diametro R Q, verticibus R, Q, ordinatâ ex T parallelâ R Y, exhibenda eſt ſectio, per prop. 1. caſ. 2. & Q V ſectionem continget.

Supple figuras horum caſuum.

Si fortè recta R M tranſit per T, Diametro R T, verticibus R, T, ordinatâ Q M, exhibenda eſt ſectio.

Si ordinata per T tranſit per S vel Q, Ductâ per R rectâ biſecante S T, vel Q T, hâc ſumptâ pro diametro, dimidiâ S T vel Q T pro ordinatâ, vertice R, & aliâ ordinatâ per Q vel S, exhibenda eſt ſectio. Haud abſimilis eſt caſus, ſi ordinata ex T vel Q tranſit per P.

Si coincidant puncta Q, S, hoc eſt, ſi detur recta Q S pro contingente in Q, coincident rectæ SRy, Q X R, rectaque Qx. *i.e.* Q S (per *coroll.* 1. prop. 35. p. 1.) continget ſectionem, quod oportuit. Idem erit de punctis Q, P, per ejuſdem prop. *coroll.* 4.

Sin coincidentibus Q, S vel Q, P, ordinata per T tranſit per Q, Vertice R, ordinatâ dimidiâ Q T, diametro rectâ per R biſecante Q T, & contingente Q S vel Q P, exhibenda eſt ſectio, per prop. 1. caſ. 3.

Si P vel S ipſi R coincidat, hoc eſt, ſi detur R P vel R S pro contingente, coincidet R Y ipſi R P vel vel R S, propter T X infinitam, vel æqualem Tx.

In omnibus his caſibus manet reliqua, tam conſtructio, quàm demonſtratio; aliquando etiam utraque evadit ſimplicior.

Cùm hæc conſtructio manifeſtè requirat, quòd (ratione munerum quæ ſingulis punctis aſſignavimus) præ-

ter P vel S nullum infinite diſtet, quódque nec P ipſi S,
nec Q ipſi R, nec T alii cuivis coincidat; ut in caſu,
ubi dantur (pro quinque punctis) duæ rectæ quæ in da-
tis punctis contingant cum infinitè diſtante puncto, di-
rectè locum non habeat; Nihilominus ad hunc quoque
caſum extendi poteſt. Nam ſi tangentes Q P, R S con-
9. currant in A; per A ductâ rectâ A B, ſecundùm infi-
nitè diſtantis puncti T directionem, ſecante tactus con-
jungentem in B, biſectâque A B in *t*; duarum contin-
gentium & puncti *t* ope exhibebitur ſectio, cujus
aſymptoto, vel diametris (ſi parabola fuerit,) erit A B
[vel P T] parallela: ſecus enim propter prop. 1. p. 2.
non biſecaretur A B à ſectione in *t*, quod, ſi parallela
ſit, omnino fiet per ejuſdem prop. *coroll.* 4. vel ejuſ-
10. dem partis prop. 2. Sin contingentes parallelæ ſint;
Erit medium punctum tactus conjungentis B ſectionis
centrum, per quod ſecundùm infinitè diſtantis puncti
directionem ductâ rectâ B C ſecante unam ex tangenti-
bus in C, erit hæc una ex aſymptotis, factâque P E =
P C, erit connexa B E altera; reditque caſus in illum
prop. præcedentis.

Si pro duobus punctis detur recta quæ in dato puncto contingat, oportet ut non per aliud ex datis punctis tranſeat, nec ſit rectis ſecundum quarum directionem aliquod ex datis punctis infinite diſtat parallela. Nam uti recta in tribus punctis ſectioni non occurrit, ita contingens ſectionem ei non ampliùs occurrit, nec poteſt eſſe aſymptoto hyperbolæ vel parabolæ diametris parallela.

Coroll. Hinc, & ex *coroll.* prop. 1. ſequitur Inveniri poſſe ſuperficies conicas numero infinitas quarum quævis propoſito ſatisfacit.

Prop. IV. Probl. IV.

11, 12. *Per data quævis quinque puncta* P, Q, R, S, T [*quorum unum, vel duo infinitè diſtare poſſunt,*] *ſectionem conicam vel ſect. opp. in plano deſcribere. Oportet autem*

Supple reliquos caſus.

autem horum nulla tria in directum jacere, nec aliquod ſecundum duo alia conjungentis directionem infinite diſtare.

Junctis *ex.gr.* P Q, P R & S Q, S R, [vel ductis ſecundum datas directiones parallelis ſi P vel S infinite diſtet,) agantur ipſis P Q, P R reſpectivè parallelæ T X, T Y, rectis Q S, R S in X, Y occurrentes, in quibus ſumantur puncta x, y vel ſimul ad eaſdem, vel ſimul ad contrarias puncti T partes, ad quas jacent X, Y, ut ſit T X : T Y : : T x : T y, & connectantur Q x, R y ſeſe in $ſ$ interſecantes, vel forte inter ſe parallelæ; Erit punctum $ſ$ ad ſectionem, vel ſaltem erunt Q x, R y hyperbolæ [vel ſect. opp.] aſymptoto, vel parabolæ diametris parallelæ. Atque hoc modo innumera puncta inveniri poſſunt.

Quòd ſectio conica [vel ſect. opp.] per puncta P, Q, R, S, T omnino tranſire poſſit, liquet per prop. præced. Et quòd unica, per *coroll.* 5. prop. 35. p. 1; Ad quam (per ejuſdem prop. *coroll.* 2.) erit punctum $ſ$, vel ſaltem erunt Q x, R y aſymptoto hyperbolæ, vel parabolæ diametris parallelæ.

Si, pro duobus punctis, detur poſitione recta quæ in dato ejus puncto contingat, (quam per aliud quodvis ex datis punctis non tranſire oportet,) Concipe puncta Q, P vel Q, S in dato illo puncto coire, & à datâ illâ rectâ conjungi; Et ſi duæ dentur ejuſmodi rectæ, concipe punctum R & reſiduum ex punctis P, S coincidere, & à ſecundâ rectâ conjungi: Et non erit conſtructio vel demonſtratio diverſa. 13. *pro omnibus caſibus.*

Si vero ſimul cum duabus contingentibus detur punctum infinite diſtans, invento novo (finite diſtante) puncto, ut in ſimili caſu prop. præced. Sectio hâc methodo deſcribi poteſt; Si vero contingentes parallelæ ſint, inventis (quemadmodum ibi oſtendimus) aſymptotis, deſcribetur per prop. ſequentem.

Coroll. 1. Hinc & ex prop. 35. p. 1. cum *coroll.* 5. liquet, Eorundem quinque punctorum, utcunque vices permutantium, ope, eandem ſemper ſectionem [vel ſect. opp.] deſcribi.

Coroll. 2. Et ſi pauciora dentur ſectionis puncta, ita tamen ut ſimul cum aliis datis indè determinari poſſint tot alia ejus puncta, ut ſint in univerſum quinque,(diametrorum parabolæ ſive aſymptoti hyperbolæ directione pro uno puncto æſtimatâ, rectâ vero quæ in dato ejuſdem puncto contingat pro duobus,) Sectio hâc methodo deſcribi poteſt.

Exempli gratiâ.

Si dentur centrum, & tria tantùm puncta ſectionis, vel duo cum contingente in eorum altero, dantur totidem ex alterâ parte centri per prop. 22. & *coroll.* 8. prop. 20. p. 1.

Similiter datis tribus punctis, vel duobus & contingente in eorum altero, datâque poſitione rectâ pro diametro cum angulo quem ad eandem faciant ordinatæ, dantur totidem ad alteras partes diametri per prop. 20. p. 1. Et ſi in hoc caſu dentur tria tantum puncta, quorum unum in ipſam diametrum incidit, dabitur contingens in hoc puncto per *coroll.* 8. prop. 20. p. 1.

Si detur magnitudine recta pro diametro tranſversâ, proindeque duo extrema ejus puncta pro verticibus, & unum præterea punctum per quod ſectio tranſeat, cum angulo ordinatarum; dabuntur duæ contingentes in verticibus per *coroll.* 8. prop. 20. p. 1.

Si, pro Ellipſi, detur tranverſa diameter unà cum ſecundâ diametro huic conjugatâ ad eandem in dato angulo inclinatâ, idem eſt caſus cum præcedenti. Pro hyperbolâ vero [vel ſect. opp.] datâ tranſversâ diametro, atque ſecundâ huic conjugatâ ſub dato angulo ad eandem inclinatâ, ductâque pro contingente huic æquali & parallelâ per utrumvis extremum tranſverſæ punctum, quæ ibidem biſecetur, Rectæ conjungentes centrum & extrema contingentis puncta erunt aſymptoti, per def. ad prop. 23. p. 1; quarum ope, & alterutrius punctorum extremorum diametri tranſverſæ, commodiùs

diùs deſcribetur ſectio per propoſitionem ſequentem.

Si pro Hyperbolâ [vel ſect. opp.] aut Ellipſi, detur tranſverſa diameter ejuſque parameter, cum angulo ordinatarum, Idem eſt caſus cum præcedenti; nam ex his datis innoteſcit ſecunda diameter huic conjugata per prop. 24. p. 1.

Pro Parabolâ, datis tribus tantùm punctis cum diametrorum directione, ductâque ſecundum hanc directionem rectâ quæ conjungentem duo quævis ex datis punctis biſecet, & per reſiduum punctum ductâ conjungenti parallelâ, Dabitur aliud punctum ex contrariâ parte diametri.

Si cum directione diametrorum parabolæ dentur duo puncta, & recta pro contingente in eorum altero; Ductâ per contactûs punctum diametro, erit caſus ſimilis præcedenti.

Datis verò rectâ pro parabolæ diametro & puncto in eodem pro vertice, cum angulo ordinatarum, & diametri parametro; Ductâ per datum punctum rectâ ſub angulo dato pro contingente, ductâque utcunque huic parallelâ diametrum ſecante pro ordinatâ: Ex abſciſſa & parametro innoteſcit (ex prop. 25. p. 1.) longitudo ordinatæ, adeoque duo alia puncta ſectionis.

Sin pro parabolâ dentur duo puncta in quibus datæ duæ rectæ (non parallelæ) ſectionem contingant; Per contingentium occurſum & medium punctum conjungentis data puncta ductâ rectâ, innoteſcit diametrorum directio per prop. 20, p. 1. ejuſque *coroll.* 3.

Redeuntque adeò hi omnes caſus in hujus prop. caſum generalem; Atque alios ſimiliter caſus eò reducere licebit.

Scholium. Hujus methodi ſectiones conicas deſcribendi facillima eſt atque expeditiſſima praxis. In rectis T X, T Y ſumptis T I, T 1 particulis in ratione T X ad T Y, notatiſque ex utrâque puncti T parte I, II, III &c. 1, 2, 3 &c. ad deſignandum T I, T 1 ſemel, bis, ter &c. ſumptas, & per correſpondentia puncta (juxta legem priùs poſitam) ductis Q I, R 1; Q II, R 2; Q III, 14. *pro omnibus caſibus.*

R3 &c. in σ, Σ, ſ, &c. concurrentibus, erunt hæc totidem puncta ſectionis deſcribendæ.

Cùm ad rectarum TX, TY puncta magis longinqua deventum fuerit quàm praxi convenit, aut rectarum QI, R1 interſectio magis obliqua evaſerit quàm ut ſatis accuratè notari poſſit, utrique incommodo mederi licet, punctorum primò datorum loco, alia & commodiora ex jam inventis adhibendo.

Innoteſcet autem, (ſi opus fuerit,) nondum deſcriptâ ſectione, cujus generis futura ſit. Nam ſi juxta *coroll.*1. prop. 35. p. 1. ad tria puncta ducantur totidem tangentes, (punctis in hanc rem vices permutantibus,) earum ope duæ diametri (per prop.20. p.1.) inveniri poſſunt; Quæ ſi ad eaſdem partes cujuſvis tactus conjungentis concurrant ad quas ipſæ tangentes, Erit ſectio Hyperbola [ſive ſectiones opp.] Quod etiam erit, ſi duo ſimul dentur puncta infinite diſtantia. Si diametri concurrant ad contrarias partes tactus conjungentis ad quas ipſæ tangentes, Erit ſectio Ellipſis; quæ etiam (pro punctorum ſitu) forte circuli circumferentia evadat. Sin parallelæ fuerint diametri, erit ſectio Parabola. Et notandum eſt, quòd, ſi detur una contingens cum tribus punctis, duas tantum novas contingentes ducere opus fuerit: Sin dentur duæ contingentes cum uno puncto, priuſquam tertia ducatur, inveniendum erit (per methodum præcedentem) novum punctum ſectionis, ad quod vel tertia illa contingens ducatur, vel quôcum prius punctum vices permutet; Secus deerit punctum quod puncti E in *fig. coroll.* 1. prop. 35. p. 1. (i. e. puncti T in *fig.* hujus prop.) munere fungatur, quod ab aliis omnibus diverſum eſſe neceſſe eſt.

Si dentur duæ contingentes parallelæ cum uno puncto, Erit ſectio Hyperbola [vel ſect. opp.] aut Ellipſis, prout punctum illud extra, intrave, parallelas jacuerit.

Methodus hæc ad caſus, ubi dantur rectæ quæ in punctis infinitè diſtantibus contingant, hoc eſt, ubi dantur tria puncta cum unâ aſymptoto, vel unum cum utrâque, non extenditur, propter duo ſaltem puncta ſecundum

cundum eandem directionem infinitè diſtantia, proindeque coincidere cenſenda,quæ, nè X,Y (unum vel utrumque) in infinitùm abeant, diverſa eſſe oportet. His verò datis, facillimè conficitur problema per propoſitionem ſequentem.

Prop. V. Probl. V.

Datis Hyperbolæ [*vel ſect. opp.*] *utrâque aſymptoto* A H, 15.
G H, *& extrà eaſdem uno puncto* B *per quod ſectio tranſeat*; *vel datâ unâ aſymptoto cum tribus punctis* B, D, F, *ità ut nec omnia tria in directùm jaceant, nec conjungens duo aliqua ſit datæ aſymptoto parallela*: *Sectionem deſcribere.*

1. Si detur utraque aſymptotos cum uno puncto B, Per B ducantur utcunque quotlibet rectæ B A, B C &c. uni aſymptotôn in A, C &c. alteri in G, E &c. reſpectivè occurrentes, fiantque G F, E D &c. ipſis A B, C B &c. reſpectivè æquales, Erunt puncta F, D &c. ad ſectionem.

Nam ſi aſymptotis A H, G H, per punctum B fiat (per prop. 2.) hyperbola [vel ſect. opp.] hæc à lineâ ſic deſcriptâ (propter *coroll.* 1. prop. 15. p. 1.) non erit diverſa.

2. Sin una tantum detur aſymptotos A H, cum tri- 15.
bus punctis B, D, F , Junctæ B D, B F occurrant datæ aſymptoto in A, C, factiſque D E = C B, F G = A B connectatur E G, occurret hæc neceſſario rectæ A C alicubi in H; nam ſi eſſet ei parallela, ob A B = F G & C B = D E, eſſet connexa D F parallela A C contra hypotheſin: Sumptâ itaque E G pro alterâ aſymptoto, caſus hic in priorem redibit.

Si ex datis tribus punctis duo coincidant, hoc eſt, ſi detur unum punctum cum rectâ quæ in dato puncto contingat quæ non ſit datæ aſymptoto parallela, Concipe duo puncta coincidere & à datâ rectâ conjungi, & non erit caſus diverſus.

15. Si ex tribus punctis unum infinitè diſtet, hoc eſt, ſi duo tantùm dentur puncta, ex. gr. B, D, cum alterius aſymptoti directione (quæ cum datæ aſymptoti directione non ſit eadem) Connexâ BD occurrente datæ aſymptoto in C, factâque $DE = CB$, per E ſecundum datam directionem ducatur EG; Hâc pro alterâ aſymptoto aſſumptâ, conſit iterum problema per caſum 1.

Prop. VI. Probl. VI.

16, 17. *Datâ pro Hyperbolæ* [*vel ſect. opp.*] *axe determinato vel majore Ellipſeos, rectâ* AB, *cum utroque foco* F, G, *ut ſit* $AF = GB$; *Sectionem, vel ſect. opp. deſcribere.*

In FG (productâ pro hyperbolâ vel ſect. opp.) ſumatur utcunque punctum C; centro utrovis foco F vel G, intervallo AC, rurſus centro reſiduo foco G vel F, intervallo CB, deſcribantur arcus circulares ſeſe interſecantes in D; Erit punctum D ad ſectionem: Eodemque modo, novorum punctorum C ope, innumera ejuſmodi puncta D inveniri poſſunt.

Junctis FD, GD, Ob $DG = AC$ vel BC, $FD = BC$ vel AC erit pro Hyperb. $FD - DG$ vel $DG - FD$, pro Ellipſi $FD + DG$, $= AB$: Si igitur tranſverſâ diametro AB, & ſecundâ huic conjugatâ ita ſumptâ ut ejus quadratum æquale ſit rectangulo $AG \times GB$, & angulo ordinatarum recto, juxta *coroll.* 2. prop. 4. fieri intelligatur ſectio [vel ſect. opp.] cui GD *ex. gr.* (ſi opus producta) occurrat in *d*, & connectatur F*d*; erunt (per prop. 1. p. 4.) Foci F, G; & (per prop. 5. p. 4.) in hyperb. vel ſect. opp. $Fd - dG$ vel $dG - Fd = AB = FD - DG$ vel $DG - FD$; in Ellipſi $Fd + dG = AB = FD + DG$; quod fieri non poteſt niſi conicidant D, *d*; & ſic ubique.

Prop.

Prop. VII. Probl. VII.

Datis Parabolæ axe A F, *vertice* A, & *foco* F; *Sectionem deſcribere.* 18.

In F A, ultra A productâ, ſumatur A B = A F, & ducatur B E ad A B perpendicularis; ſumptoque in A F, ultra A, puncto C, centro C intervallo F C deſcriptus arcus ſecet B E in E, deinde centris E, F, eodem intervallo deſcripti arcus ſeſe interſecent in D; Erit punctum D ad ſectionem; Eodemque modo innumera ejus puncta inveniri poſſunt.

Eſt enim quadrilaterum F C E D (per conſtruct.) æquilaterum, adeoque parallelogrammum, unde E D parallela A F, & F D = D E. Si jam diametro B A F, vertice A, parametro = 4 A F, angulo ordinatarum recto, fieri intelligatur (juxta *coroll.* 2. prop. 4.) parabola, atque huic (ſi opus producta) occurrat E D in *d*, & connectatur F *d*, erit (per prop. 2. p. 4.) F focus, & (per prop. 10. p. 4.) E *d* = F *d*; connexâque E F, erunt ob commune latus E F & communem angulum F E D, triangula iſoſcelia F D E, F *d* E invicem æqualia, hoc eſt coincident D, *d*; Et ſic ubique.

Prop. VIII. Probl. VIII.

Datis duabus rectis F N, F A *ſeſe in puncto* F *interſecantibus, & in earum alterâ alio puncto* A; *Hyperbolam [vel ſectiones oppoſitas] deſcribere, cujus focus alter ſit* F, *recta* F N *axis, recta* F A *aſymptoto parallela, & punctum* A *ad ſectionem*; *Oportet autem ut angulus* N F A *non ſit rectus.* 19.

In rectâ F A, ultra A, ſumatur A B = F A, & ex B demittatur in F N perpendicularis B N; In F A, ex utravis parte puncti F, ſumpto puncto C, ita tamen ut ſit C F major perpendiculari à C in rectam B N dimiſsâ, Centro C, intervallo CF, deſcribatur arcus circuli ſecans BN in E, rurſus centris E, F eodem intervallo deſcripti arcus ſeſe interſecent in D; Erit punctum D ad ſectionem; Eodemque modo quotlibet ejuſmodi puncta inveniri poſſunt.

Ductâ AL parallelâ FN ſecante NB in L, dividatur NF in K, ut ſit FA : AL :: FK : KN, productâque FN fiat

$$1.\ FK - KN : KN :: 2FK : KI,$$

productâque FI fiat IG = KF; Focis F, G, axe tranſverſo IK, (juxta prop. 6.) fiant ſectiones oppoſitæ, quarum uni occurrat FA in *a*, & ducatur *al* parallela FK ſecans NB in *l*. Propter proport 1. erit componendo.

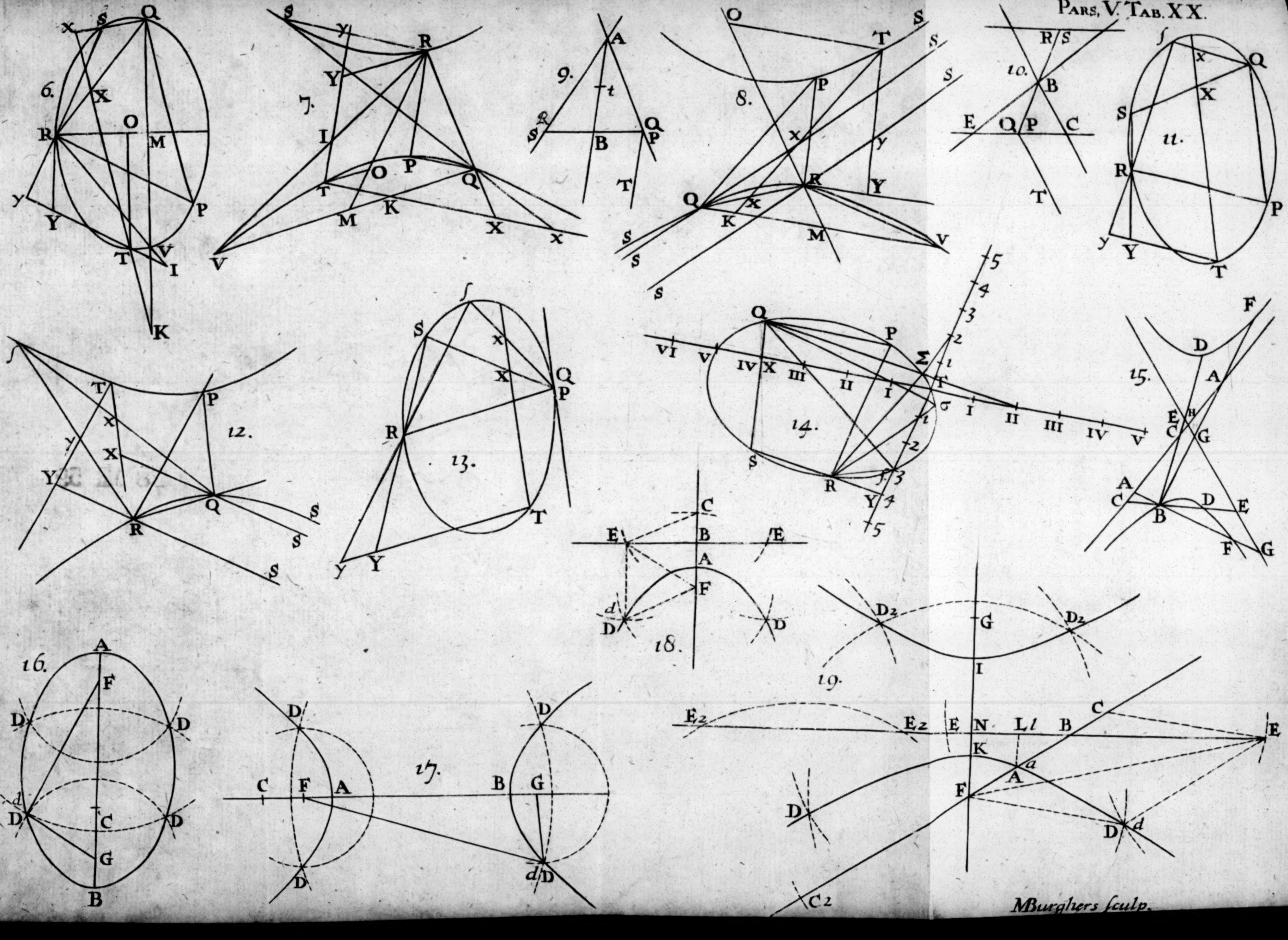
$$2.\ FK : KN :: \left\{\begin{matrix} 2FK + KI \\ GF \end{matrix}\right\} : KI ::$$

(per prop. 9. p. 4.) F*a* : *al* :: (prius) FA : AL; Conicidunt itaq; puncto A, *a*; Nam ſi dicatur F*a* $\sqsubset$ FA (cùm NL, FA ultra L, A productæ coeant in B) erit LA $\sqsubset$ *la* contrà quàm oportuit, & vice versâ; Unde punctum A erit ad ſectionem. Rurſus per proport. 2 alternando & dividendo

$$\left.\begin{matrix} GF - FK \\ IF \end{matrix}\right\} : FK :: \left\{\begin{matrix} KI - KN \\ IN \end{matrix}\right\} : KN,$$

erit itaq; (propter *Coroll* 1. prop. 1. p 2.) punctum N illud in quod coeunt tangentes in extremis ordinatæ ad diametrum KF per F ductæ; quæ ordinata (cùm KF axis ſit) erit parallela NB, utpote ad KF perpendicularis; Eritq; adeo AF aſymptoto parallela; ſecus enim occurreret ſectioni vel ſect. opp. in binis punctis, & (propter prop. 3. p. 2.) non eſſet FA = AB contrà quàm fieri præceptum eſt. Quòd vero punctum D ſit ad ſectionem, eodem modo liquet quo (prop. præced.) in parabolâ.

28 MR 59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PARS VI.

Lemma.

Sectionibus oppoſitis & Ellipſi, ſit diameter quævis 20. 21.
(quæ in ſect. opp. ſit determinata) A B, *huic conjugata*
KCE, *ſemidiameter ſecunda ſemidiametro* C A *con-*
jugata CM; *ſi recta* TK *ſectionem vel utramvis*
ſect. opp. in T *contingens occurrat* CEK *in* K, *& à*
tactu T *agatur* TE *ipſi* AB *parallela occurrens*
KCE *in* E; *Dico* CK : CM : CE ∺ *i. e.* CK ×
CE = CMq.

N Ellipſi jam patet per coroll. 2. prop. 1. p. 2. generaliter vero ſic.

Ductæ AO, BL parallelæ CKE, adeoque ſectio-
m vel utramque ſect. in A, B contingentes, occurrant
ntingenti TK in O, L, & ducatur TF his parallela
ccurrens AB in F; Erit ob ſimiliia triangula
AO : FT :: DA : DF, &
CK : BL :: DC : DB; ſed (per cor. 1. prop. 1. p. 2.)
DA : DF :: DC : DB, ergo
AO : FT :: CK : BL; unde AO × BL = (per
roll. 1. prop. 10. p. 2.) CMq = CK × FT = (ob re-
as parallelas) CK × CE.

Prop. I. Theor. I.

a Sectionibus opp. & Ellipſi, ſi recta quævis DTK 22. 23.
ſectionem vel utramvis ſect. opp. in T *contingens dua-*
bus quibuſvis diametris conjugatis ECM, ACB, *ſi*
opus productis, in K, D, *occurrat*; *Dico* DT × TK

$=\frac{1}{4}$ *fig. diam. per* T; *i. e.* (*ſi* CG *ſit ſemidiameter ſecunda ipſi* CT *conjugata*) $=$ CGq.

Sectionem vel ſect. opp. in A, B contingant AO, BL contingenti DTK occurrentes in O, L; & à T ipſis AO, ECM, BL, parallela (i. e. ad diametrum AB ordinata) ducatur TF, occurrens AB in F; diameter AB (per coroll. 1. prop. 1. p. 2.) harmonicè ſecatur in punctis D, A, F, B; unde, propter parall. rectas AO, TF, BL, recta DTK, (per lemm. 8. p. 2.) harmonicè dividitur in D, O, T, L; ſed & propter parall. rectas, & AC $=$ CB, erit OK $=$ KL; unde (per lemm. 2. vel 3, p.2.) TK : TL :: OT : DT; unde TK×DT $=$ TL×TO $=$ (per coroll. 2. prop. 10. p. 2.) $\frac{1}{4}$ fig. diam. per T $=$ CGq.

Prop. II. Theor. II.

24. 25. *In Ellipſi ſumma, in Oppoſitis ſectionibus differentia, quadratorum ſemidiametri cujuſvis* TC *& ſecundæ huic conjugatæ* CG *æqualis eſt ſummæ, vel differentiæ, quadratorum ſemiaxis* AC & *ſecundi huic conjugati* CM.

Recta in T contingens occurrat axibus (ſi opus productis) in D, K, & ex T ſint ad axes ordinatæ TL, TF; In Ellipſi

$$KDq=\left\{\begin{matrix}KTq\\ KLq+TLq\\ KLq+FCq\end{matrix}\right\}+\left\{\begin{matrix}TDq\\ TFq+DFq\\ LCq+DFq\end{matrix}\right\}+2TK\times TD$$

Rurſus

$$KDq=\left\{\begin{matrix}KCq\\ KLq+LCq+2KL\times LC\end{matrix}\right\}+\left\{\begin{matrix}CDq\\ CFq+DFq+2CF\times DF\end{matrix}\right\};$$

unde demptis utrinque æqualibus, & reſidua dimidiando, TK × TD $=$ KL × LC $+$ DF × FC; Sed per prop. præced. TK × TD $=$ CGq $=$ KL × LC $+$ DF × FC; Eſt vero $CTq=CLq+\left\{\begin{matrix}TLq\\ CFq\end{matrix}\right\}$ ergo

CGq

$$CGq+CTq=\left\{\begin{matrix}KL\times LC+LCq\\ CK\times CL\end{matrix}\right\}+\left\{\begin{matrix}DF\times FC+FCq\\ CF\times CD\end{matrix}\right\}$$

$=$ (per coroll. 2. prop. 1. p. 2.) $CMq+CAq$.

In Sectionibus oppoſitis

$$TKq=\left\{\begin{matrix}TDq\\ TFq+FDq\\ LCq+FDq\end{matrix}\right\}+\left\{\begin{matrix}DKq\\ DCq+CKq\end{matrix}\right\}+2TD\times DK;$$

Rurſus

$$TKq=\left\{\begin{matrix}TLq\\ FDq+DCq+2FD\times DC\end{matrix}\right\}+\left\{\begin{matrix}LKq\\ LCq+CKq+2LC\times CK\end{matrix}\right.;$$

unde $TD\times DK=FD\times DC+LC\times CK$;
& addendo utrinque TDq,

$$\left.\begin{matrix}TD\times DK+TDq\\ TD\times TK\\ CGq\end{matrix}\right\}=FD\times DC+LC\times CK+\left\{\begin{matrix}TDq\\ LCq+FDq;\end{matrix}\right.$$

Eſt vero $TCq=\left\{\begin{matrix}TFq\\ LCq\end{matrix}\right\}+\left\{\begin{matrix}FCq\\ FDq+DCq+2FD\times DC;\end{matrix}\right.$

& deletis utrinque æqualibus, erit quadratorum CGq TCq differentia eadem quæ reſiduorum $LC\times CK$, & $\left\{\begin{matrix}DCq+FD\times DC\\ FC\times DC\end{matrix}\right\}$ i. e. (per coroll. 2. prop. 1. p. 2. & lemma præc.) ipſorum CMq, CAq.

Coroll. Hinc in Ellipſi ſumma, in Sectionibus oppoſitis differentia, quadratorum ſemidiametri cujuſvis ejuſque conjugatæ, eſt alterius cujuſvis diametri ejuſque conjugatæ quadratorum ſummæ vel differentiæ æqualis.

Prop. III. Theor. III.

In Hyperbola ſive ſect. opp. et Ellipſi, ſi recta DTK *ſectionem vel utramvis ſectionum opp. in* T *contingat, & erecta ad contingentem perpendicularis* TV *utrique axi in* V, Y *occurrat; erit* $TV\times TY=\frac{1}{4}$ *fig. diametri per* T. 26. 27.

Contingens occurrat axibus (ſi opus productis) in K, D; propter angulos rectos DTV, DCK, & angu-

lum

lum CDK communem, erit triang. DTV triang. DCK ſimile; & ob ang. rectos YTK, DCK, & YKT communem, triangulum YTK ſimile erit triang. DCK; ergo ſimilia ſunt triang. DTV, YTK, unde DT : TV : : YT : TK hoc eſt $TV \times YT = DT \times TK =$ (per prop. 1.) $\frac{1}{4}$ fig. diam. per T.

28. 29. *Coroll.* 1. Poſitis quæ in figuris præcedentibus, ſit CG ſemidiameter ſecunda ipſi TC conjugata, erit $TV \times TY = CGq$; i. e. TV : CG : TY ∺, & ratio TV ad TY duplicata rationis TV ad CG, ſive CG ad TY, adeoque cum ſit (per coroll. prop. 9. p. 4.) TV ad CG, ut ſemiparameter axis majoris vel determinati ad ſemiaxem ſecundum huic conjugatum, erit ubique TV ad TY, ut eadem ſemiparameter ad ipſum ſemiaxem majorem ſive determinatum.

Coroll. 2. Si ex puncto Y ducatur YE faciens cum recta TY angulum $TYE = TCV$, occurrenſque diametro per T in E, erit $TE = \frac{1}{2}$ param. diametri TC. Nam ob ſim. triang. TCV, TYE, TV : TC : : TE : TY, unde $TV \times TY$ ($= \frac{1}{4}$ fig diam. per T) $= TC \times TE$, ergo $TE = \frac{1}{2}$ param. diam. per T.

Coroll. 3. Ideoque ex E erecta ad TE perpendiculari EF, quæ occurrat TY in F, erit circulus centro F intervallo FT deſcriptus ſectioni in puncto T æquicurvus. Abſcindet enim ex TC (ſi opus producta) $TQ = 2\,TE =$ parametro diametri per T; adeoque (per coroll. 1. prop. 11. p. 3.) erit ſectioni in puncto T æquicurvus.

Prop. IV. Theor. IV.

28. 29. *In Hyperbola ſive ſect. opp. & Ellipſi; Dico ſolidum ex rectangulo ſemiaxium in radium circuli ſectioni in dato puncto* T *æquicurvi, viz.* $AC \times MC \times TF$, *æquale eſſe cubo ex ſemidiametro* CG *diametro per* T *conjugata, i. e.* CG^3.

Manentibus quæ in prop. præced. & ejus corollariis;

producta (ſi opus) CG occurrat TV in D, erit ob ſim. triang. $TD : TC :: TE : TF$, unde $TC \times TE = TD \times TF$; ſed (per prop. 11. p. 2.) erit $TD \times CG = AC \times MC$ hoc eſt $\frac{AC \times MC}{CG} = TD$; unde $TC \times TE = \frac{AC \times MC}{CG} \times TF$; i.e. $\left\{\frac{TC \times TE \times CG}{CG^3}\right\} = AC \times MC \times TF$.

Coroll. 1. Hinc $\frac{CG^3}{AC \times MC} = TF$; & ob datum quantitatem $AC \times MC$, erit TF ubique ut CG^3.

Coroll. 2. Si parameter axis majoris vel determinati dicatur L, cum ſit (per coroll. prop. 6. p. 4.) $TV : CG :: \frac{1}{2}L : CM$, adeoque $TV^3 : CG^3 :: \frac{1}{8}L^3 : CM^3$, erit $\frac{TV^3 \times CM^3}{\frac{1}{8}L^3} = CG^3 =$ ut prius $AC \times MC \times TF$; hoc eſt $\frac{TV^3 \times CM^2}{\frac{1}{8}L^3} = AC \times TF$, vel (propter $\frac{CM^2}{\frac{1}{2}L} = AC$) $\frac{TV^3}{\frac{1}{4}L^2} = TF$; & ob datam quantitatem $\frac{1}{4}L^2$, erit TF ubique ut TV^3.

Coroll. 3. Similiter, cum ſit $TV : TY :: \frac{1}{2}L : AC$ adeoque $TV^3 : TY^3 :: \frac{1}{8}L^3 : AC^3$, erit $TV^3 = \frac{TY^3 \times \frac{1}{8}L^3}{AC^3}$, & $\frac{TV^3}{\frac{1}{4}L^2} = \frac{TY^3 \times \frac{1}{2}L}{AC^3} = TF$; & ob datas quantitates $\frac{1}{2}L$, & AC^3, erit TF ubique ut TY^3.

Prop. V. Theor. V.

In parabola ſit AV *axis,* TV *perpendicularis in tangentem in puncto quovis* T, *ſecans axim in* V, *Radius circuli ſectioni in puncto* T *æquicurvi* TF, *&* *ſemiparameter axis dicatur* $\frac{1}{2}L$; *Dico ſolidum ex qua-* 30.

quadrato ſemiparametri axis in radium circuli æquicurvi TF *æquale eſſe cubo ex* TV; *hoc eſt* $\frac{1}{4}L^2 \times TF = TV^3$.

Circulus ſectioni in puncto **T** æquicurvus (per prop. 11. p. 3.) abſcindet à diametro per T rectam TQ æqualem parametro ejuſdem diametri, quam perpendicularis ex centro F biſecabit in E; dimiſſaque in axem ex T perpendiculari TB, erit (per prop. 14. p. 3.) $BV =$ ſemiparam. axis $= \frac{1}{2}L$; unde (per prop. 15. p. 3.) $TE = BV + 2AB$.

Ob ſim. triang.

$$TV : BV :: TF : \left\{\begin{matrix} TE \\ BV + 2AB, \end{matrix}\right. \text{unde}$$

$$TV \times BV + TV \times 2AB = BV \times TF;$$

Sed (per prop. 15. p. 1.) $AB \times L = 2AB \times \frac{1}{2}L =$

$$2AB \times BV = TBq = TVq - BVq, \text{unde } \frac{TVq - BVq}{BV} = 2AB;$$

$$\text{Ergo } TV \times BV + \frac{TV^3 - TV \times BVq}{BV} \text{ id eſt}$$

$$\frac{TV \times BVq + TV^3 - TV \times BVq}{BV} = BV \times TF; \text{ vel}$$

$$TV^3 = BVq \times TF = \frac{1}{4}L^2 \times TF.$$

Coroll. Hinc $\frac{TV^3}{\frac{1}{4}L^2} = TF$, & ob datam quantitatem $\frac{1}{4}L^2$, erit TF ubique ut TV^3.

Prop. VI. Theor.. VI.

31. 32. *In Hyperbola ſive ſect. opp. & Ellipſi, ſectionem in* T *contingat recta* TDK, *& à tactu* T *erigatur perpendicularis in contingentem* TVY, *minori axi in* Y *occurrens, ductâque ad utrumvis focum* F *rectâ* TF, *& à puncto* Y *in* TF (*ſi opus productam*) *perpendiculari* YS; *Dico* TS *æqualem eſſe ſemiaxi majori ſive determinato* AC.

Ex puncto V ubi TY majori axi occurrit ducatur ad TF vel TH perpendicularis VR, eſtque (per coroll. 1. prop. 3.) $TV : TY :: \frac{1}{2}$ param. axis : $CA ::$ (ob ſim. triang.) $TR : TS$; ſed (per prop. 6. p. 4.) $TR = \frac{1}{2}$ param. axis, ergo & $TS = CA$.

Coroll. 1. Ducta ex T ad axem CM ordinata TNG ſectioni denuo, vel ſect. opp. in G occurrente, Liquet circulum centro Y radio TY deſcriptum rectam TK, adeoque ſectionem, in T contingere, & ob ang. TMC rectum, & $TN = NG$, eandem denuò vel ſect. opp. in G contingere; & porro à recta TF per T & utrumvis focum F ducta abſcindere rectam TL axi AB æqualem, utpote ipſius TS duplam. Idem erit in recta THI occurrente circulo in I.

Coroll. 2. Unde, & ob $TF \pm TH$ vel $TH - TF$ = (per prop. 5. p. 4.) $2CA = AB$, erit $TH = FL$, & $TF = HI$, & $TH \times HI = TF \times FL = TH \times TF = \frac{1}{4}$ fig. diam. per T.

Coroll. 3. Cum (per coroll. 1. prop. 5. p. 4.) ductâ ex centro C ad contingentem uſque rectâ CO parallelâ FT, ſit $CO = AC = TS$, liquet connexam CS eſſe parallelam contingenti TK, & proinde diametrum diameteo per T conjugatam; vel quod idem eſt, diametrum diametro per T conjugatam, & perpendicularem ex Y in rectam TF, cum eadem recta TF coire in uno eodemque puncto S.

Coroll. 4. Producta ſemidiametro TC donec ſectioni vel ſect. opp. occnrrat denuo in P, ob $TS = SL$, & $TC = CP$, liquet connexam LP parallelam eſſe DK, & CS, proindeque ſectionem in P contingere.

Prop. VII. Theor. VII.

In Hyperbola ſive ſect. opp. & Ellipſi, poſitis quæ in 33. 34
præcedentibus, ſi à puncto Y *ducatur ad utrumvis focum* F *recta* YF; *Dico* TY *eſſe ubique ad* YF, *ut axis major vel determinatus* AB *ad diſtantiam focorum* HF, *vel ut* AC *ad* HC, *vel* CB *ad* CF.

Nam ob T V × T Y = $\frac{1}{4}$ fig. diam. per T = per (coroll. 2. prop. 5. p. 4.) T H × T F, erit T H : T V : : T Y : T F, unde ob æquales angulos H T V, V T F ſimilia erunt triangula T H V, T Y F, & ang. T H V = T Y F, unde

TH : HV : : TY : YF; ſed propter ang. HTV = YTF, erit

$TH:HV::TF:FV::\left\{\begin{matrix}TF\pm TH\\ AB\end{matrix}\right\}:\left\{\begin{matrix}FV\pm HV\\ HF\end{matrix}\right\}::CB:CF$

: : (prius) T Y : Y F.

Pari modo ob ſimilia triangula T F V, T Y H inſtitui poteſt demonſtratio in recta Y H.

Prop. VIII. Theor. VIII.

33. 34. *In Hyperbolâ ſive ſect. opp. & Ellipſi, poſitis quæ in præcedentibus, ſi producantur* Y H, Y F *donec occurrant circulo radio* T Y *deſcripto in* N, M, *& conjungantur* N A, M B; *Dico* N A, M B *ſectionem vel ſect. opp. in* A, B *contingere, & angulum* A N H *vel* B M F *æqualem eſſe angulo* D T H *ſive* K T F.

Cum ſit (per præced.) C B : C F : : T Y = Y M : Y F, erit MB parallela C Y, proindeque ſectionem vel unam è ſectionibus continget in B ; & pari ratione N A ſectionem vel alteram è ſectionibus continget in A.

Ob ſim. rectang. triang. C H Y, C F Y, B F M, A H N, & Y F = Y H, Y N = Y M, erit N A = M B.

Rectæ Y H, Y F, ſi opus productæ occurrant circulo in O, L; propter C B : C F vel C H : : T Y = Y O : Y F, vel Y H, erit connexa O B parallela C Y, ſectionemque in B continget; parique ratione erit L A parallela C Y, & continget in A.

Una itaque eademque recta erunt M B, B O, & ſimi-
L liter N A, ~~M~~ A.

Pari etiam ratione qua N A, M B, erunt B O, A L æquales, unde erit connexa L O = axi A B.

Recta T H vel T F, ſi opus producta occurrat circulo in P, erit (per coroll. 1. prop. 6.) T P = A B = L O, adeoque arcus T P = L O ; unde angulus D T P i.e. D T H ſive K T F = L N O, vel L M O i.e. A N H vel F M B.

Prop. IX. Theor. IX.

Si alterutram è ſect. opp. vel Ellipſin ubivis contingat T D *in* T, *tactum vero* T *&* *utrumvis focum* H *jungat* H T, *anguluſque* H I Z *qui idem ſit cum angulo* H T D *ita moveatur, ut altero ejus crure* H I *perpetuo per* H *tranſeunte, punctum* I, *à puncto* T *ad utramvis partem progrediendo, circuli radio* T Y (*juxta præcedentia*) *deſcripti circumferentiam percurrat; Dico crus alterum* I Z *in quovis ejus ſitu ſectionem vel unam è ſect. opp. contingere.* 35. 36.

Connexa Y H, & ſi opus producta, occurrat circulo in N, O, & connectantur N A, B O, quæ (per prop. 8.) ſectionem vel ſect.opp. in A & B contingent, eruntque parallelæ.

Quoniam angulus H I Z (ex hyp.) = H T D = (per prop. 8.) H N A, dimiſſa ex H in I Z perpendiculari H Z, erit ob ang. N A H rectum, triangulum H I Z triangulo H N A æquiangulum, & ang. I H Z = ang. N H A; unde N H : H A : : I H : H Z; connexis A Z, Z B, & N I, I O, erit ob I H Z = N H A (addito demptove communi ang. A H I vel &c.) ang. N H I = A H Z; unde, & ob N H : H I : : H A : H Z, erunt triang. N H I, A H Z ſimilia, & ang. N I H = A Z H.

Rurſus ob ang. N H I = A H Z, vel horum complementa I H O = Z H B, & ob N H : H A : : H O : H B : : H I : H Z, ſimilia erunt triang. I H O, Z H B, & ang. H I O = H Z B.

Jam ob N I H = A Z H, & H I O = H Z B, erit ipſorum N I H, H I O ſumma vel differentia, ipſorum A Z H, A Z B ſummæ vel differentiæ æqualis; ſed ipſorum N I H, H I O ſumma vel differentia i. e. ang. N I O (cum NO ſit diameter) = Recto, ergo & ipſorum AZH, HZB ſumma vel differentia i. e. AZB = Recto; unde punctum Z eſt ad ſemicirculum diametro AB deſcriptum. Et cum ang. HZI (ex conſtructione) rectus ſit, recta IZ (per prop. 1. p. 4.) non diverſa erit à contingente per Z tranſeunte, & proptereà ſectionem vel unam è ſect. opp. alicubi in X continget.

Coroll. Si recta HI ex primo ſitu HT circa H rotante, & circulum in I ſecante, ex puncto I prodeat, ad eaſdem ſemper partes rectæ IH ad quas TD ſub initio, contingens recta IZ, erit ſemper ang. HIZ = HTD. Nam recta ex I ſub angulo = HTD erit contingens, & ex eodem puncto ad eaſdem partes non prodeunt duæ contingentes.

Prop. X. Theor. X.

37. *Si Parabolam ubivis in* T *contingat recta utrinque infinita* TK, *tactum vero* T *&* *focum* H *jungat* HT, *anguluſque* HID *qui idem ſit cum ang.* HTK *ita moveatur, ut altero ejus crure* HI *perpetuò per* H *tranſeunte, punctum* I, *à puncto* T *ad utramvis partem progrediendo, rectam* TK *percurrat*; *Dico crus alterum* ID *in quovis ejus ſitu ſectionem contingere.*

Axis AH productus occurrat contingenti TK in C, & recta AK contingens in axis vertice A occurrat contingenti TI in K, ductâque HK, erit ang. HKI (per coroll. 10. prop. 2. p. 4.) rectus, & KC = TK, ideoque angulus HTK = KCA = HKA, & KHT = KHC = CKA.

Perpendicularis ex foco H in rectam ID occurrat ei in D, connexaque DK & ſi opus producta occurrat axi in E.

Ob angulos æquales HTK, HID, & HKT, HDI rectos, erunt triangula HTK, HID ſimilia, ideoque ang. IHD = THK, & HI : TH :: HD : HK; Sed ob ang. IHD = THK, erit (addito demptove communi ang. &c.) ang. I H T = D H K; unde (propter præcedentem analogiam) triangula I H T, D H K ſimilia erunt, & ang. H T I = H K D.

Si itaque puncta I, K ſint ad eaſdem partes puncti T, erit $\left\{\begin{matrix}HTI\\KCA\\HKA\end{matrix}\right\} = \left\{\begin{matrix}HKD\\HKE\end{matrix}\right\}$ ſi ad diverſas, erunt æqualium H T I, H K D complementa ad duos rectos æqualia, i. e. $\left\{\begin{matrix}HTK\\KCA\\HKA\end{matrix}\right\} = HKE$, hoc eſt erunt K A, K E una recta, proindeque, propter ang. H D I rectum, recta D I (per prop. 2. p. 4.) non diverſa erit à contingente per D tranſeunte, adeoque ſectionem alicubi in L continget.

Coroll. Si recta H I ex primo ſitu H T circa H rotante, & rectam T C in I ſecante, ex puncto I prodeat, ad caſdem ſemper rectæ H I partes ad quas T K ſub initio, recta I D quæ ſectionem contingat, erit ſemper angulus H I D = H T K.

Patet ut coroll. præcedentis.

F I N I S.

ADDENDA.

Pag. 23. lin. 30. lege, *parallelæ ſunt*, [*vel ſi* BI, CI *parallelæ ſint, erit per coroll.* 3. *&* 6. *prop.* 15. BT = VC, *adeoque* BC *parallela* TV] *unde &c.*

Pag. 31. lin. 17. *ordinata, vel ſaltem ſit contingens ordinatis parallela, habeatque &c.*

Ibid. lin. 19. *recta per centrum intra &c.*

Pag. 82. lin. 3. *ſectionem, cum recta circulum in* N *vel* O *contingens ex utraque contactus parte extra ſectionem neceſſario caderet, i. e. ſectionem etiam contingeret, ex tribus &c.*

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CORRIGENDA.

Pag. 2. lin. 18. pro *diverſo* lege *diverſum*. ibid l. 20. pro BC, lege AC p. 4. l. ult. b AB. p. 10. l. 3. AMN. p. 11. l. 10. AD. ib. l. 24. DAE. p. 32. in marg. 92. 93. p. 33. l. 7. dele, *Idem in Hyperbolâ intellige*. p. 40. l. 26. prop. 15. p. 43. l. 13. EH : EN. p. 44. l. 36. BA, CA eſſe. p. 47. l. 21. AC. p. 51. l. 29. pro FG, FGH. p. 107. dele integrum paragraphum à lin. 17. uſque ad 21, incluſivè, ut ſuperfluum. p. 114. l. 24. & 25. 4 AG×GB.

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p. 40.
p. 47.
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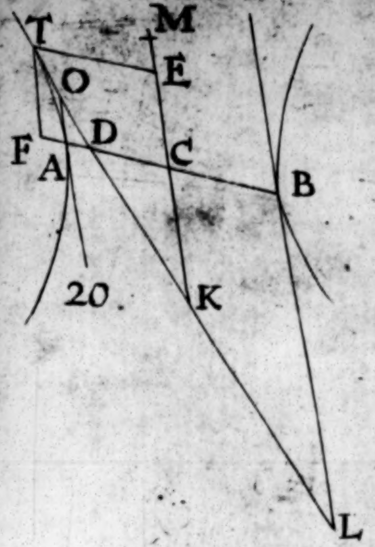

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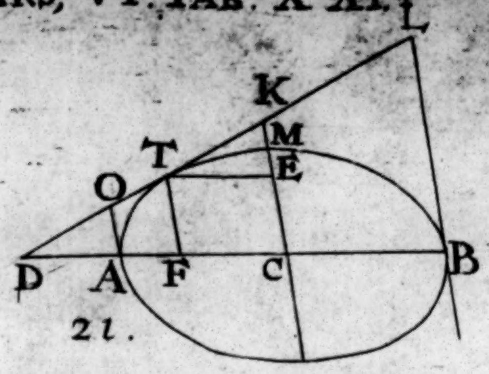

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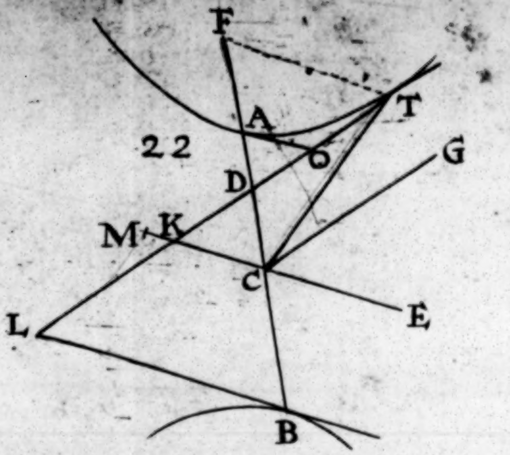

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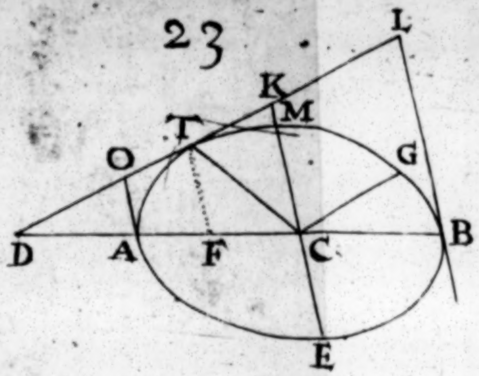

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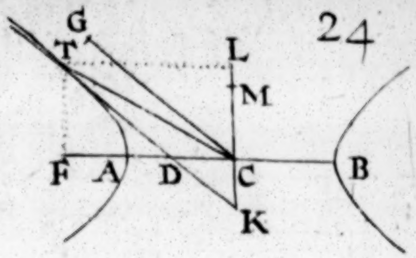

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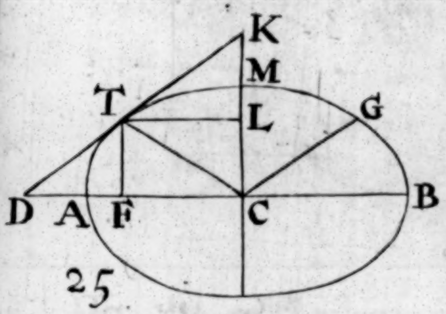

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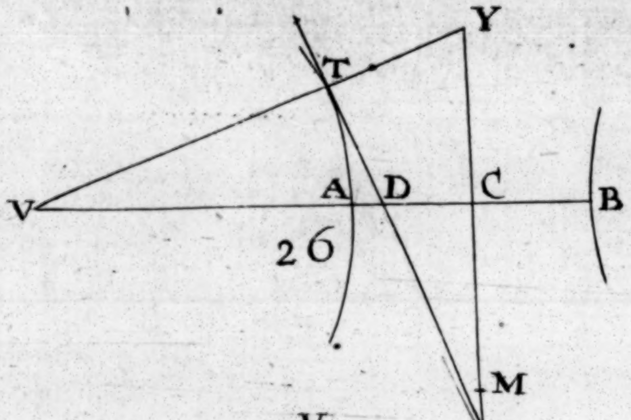

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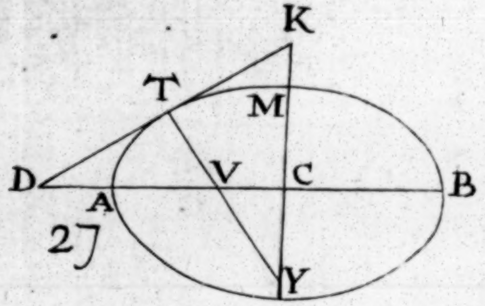

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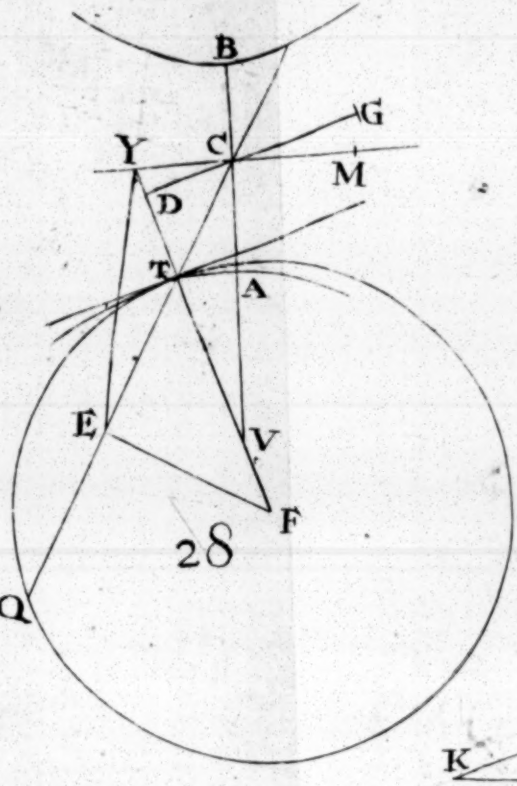

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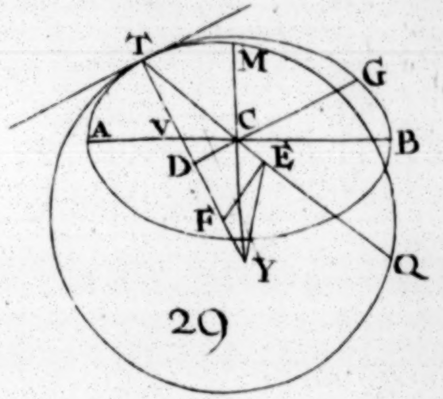

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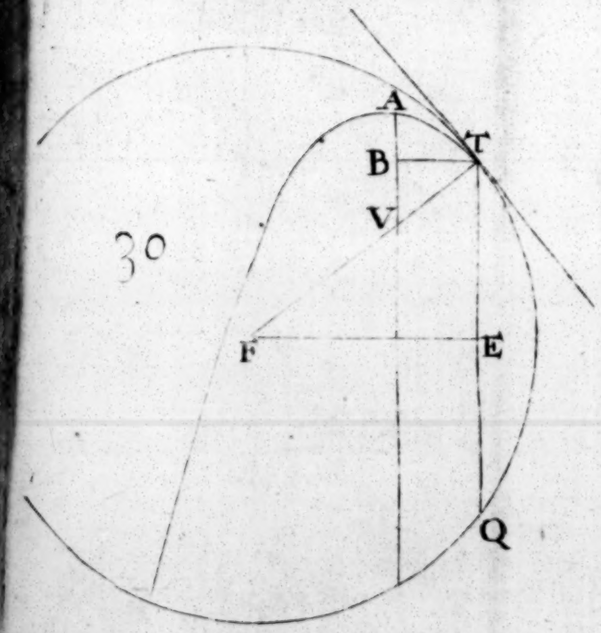

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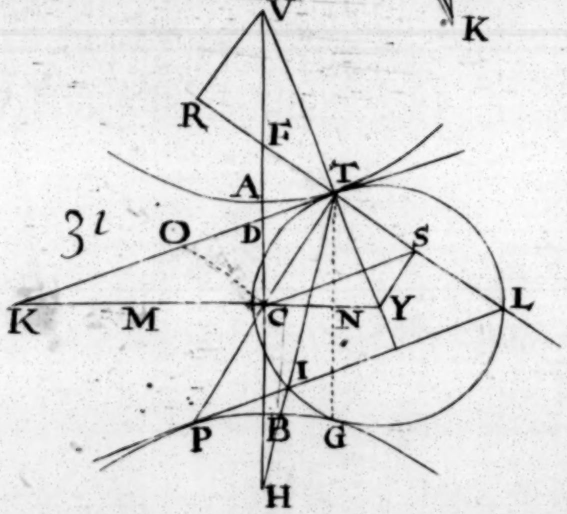

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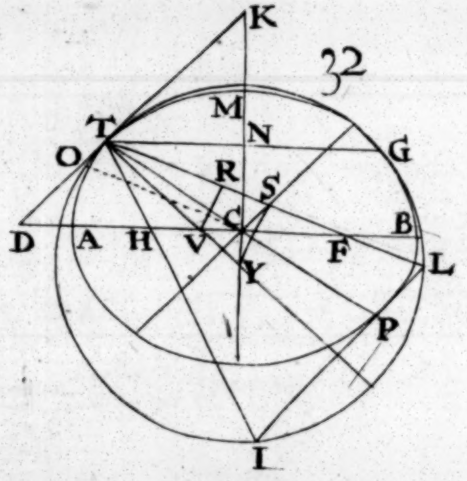

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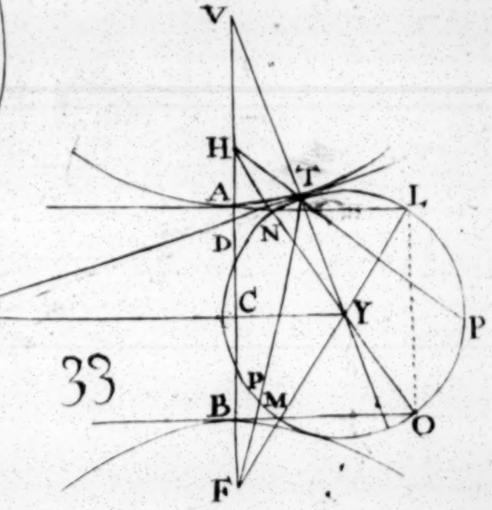

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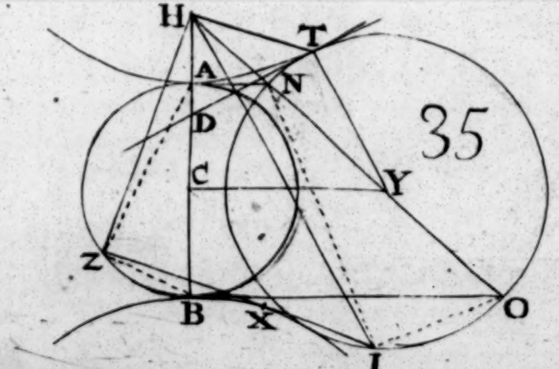

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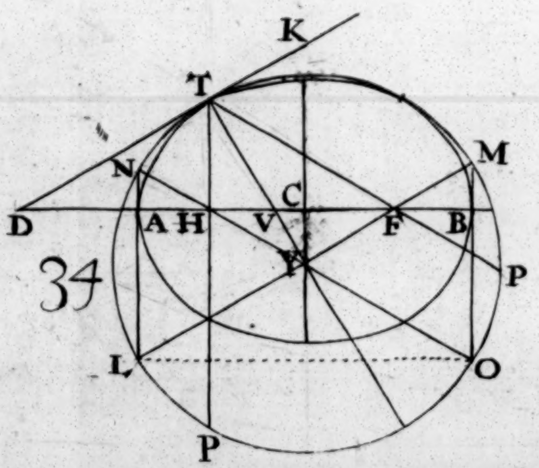

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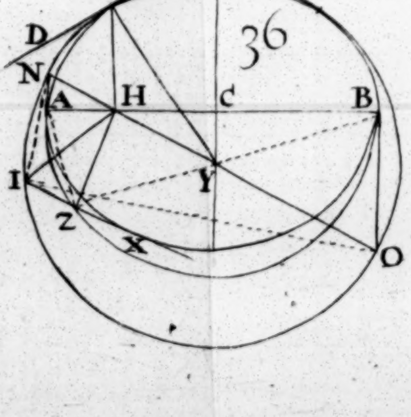

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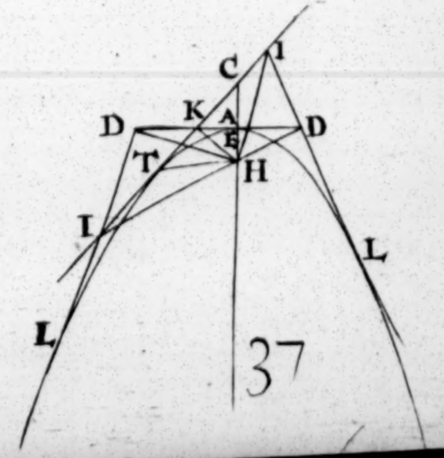

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MBurg. sculp.